QFT

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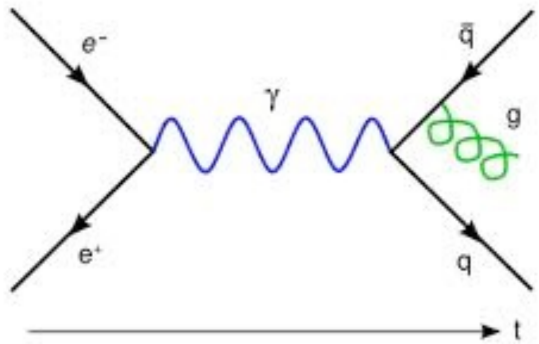
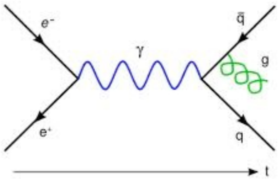
(Notes based on Dr A. Moss' lectures)



Lecture10: Interacting Dirac Field - Feynman Diagrams

Nucleon-Anti-Nucleon Scattering



- $\psi\bar{\psi} \to \psi\bar{\psi}$: Initial and final state contains a nucleon-anti-nucleon pair $|i\rangle = b^{s_1\,\dagger}(\mathbf{p}_1)c^{s_2\,\dagger}(\mathbf{p}_2)|0\rangle$, $|f\rangle = b^{r_1\,\dagger}(\mathbf{q}_1)c^{r_2\,\dagger}(\mathbf{q}_2)|0\rangle$

- Contribution to S-matrix at $O(g^2)$

$$\frac{(-ig)^2}{2}\langle 0| \int d^4x\, d^4y\, c^{r_2}(\mathbf{q}_2)b^{r_1}(\mathbf{q}_1)T\left\{\bar{\psi}(x)\,\psi(x)\,\phi(x)\bar{\psi}(y)\,\psi(y)\,\phi(y)\right\} b^{s_1\,\dagger}(\mathbf{p}_1)c^{s_2\,\dagger}(\mathbf{p}_2)|0\rangle$$

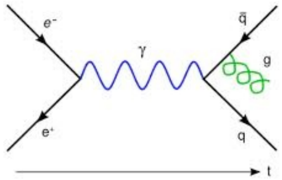
- As in bosonic case only term which contributes in time-ordered product is

$$:\bar{\psi}(x)\,\psi(x)\bar{\psi}(y)\,\psi(y): \Delta_F^{\phi}(x-y)$$

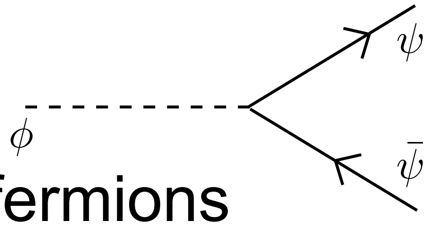
- Have to be careful with spinor indices - calculation is quite tedious (try it!)

QFT

Feynman Rules

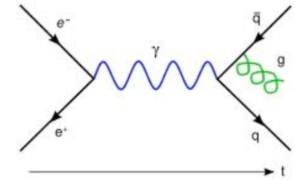


- Draw an external line for each particle in the initial and final state (as before will choose dotted lines for mesons, solid lines for nucleons)
- Add an arrow to nucleons to denote charge (incoming arrow for ψ in initial state)
- Join lines by trivalent vertices



- Associate spinors with external fermions
  - For incoming nucleon $u^s(\mathbf{k})$
  - For outgoing nucleon $\bar{u}^s(\mathbf{k})$
  - For incoming anti-nucleon $\bar{v}^s(\mathbf{k})$
  - For outgoing anti-nucleon $v^s(\mathbf{k})$

Feynman Rules



- For each vertex $(-ig)(2\pi)^4\delta^4\left(\sum_i k_i\right)$ where momenta are into vertex

- For each internal line integrate the propagator

ϕ

$$\int \frac{d^4k}{(2\pi)^4}\frac{i}{k^2 - m^2 + i\epsilon}$$

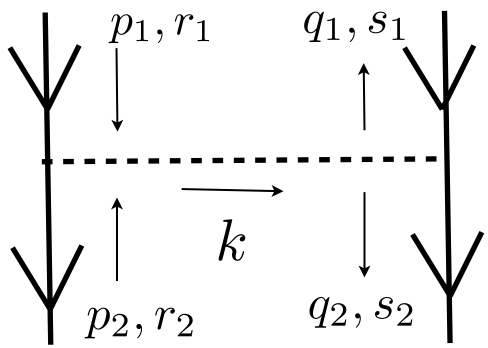
ψ

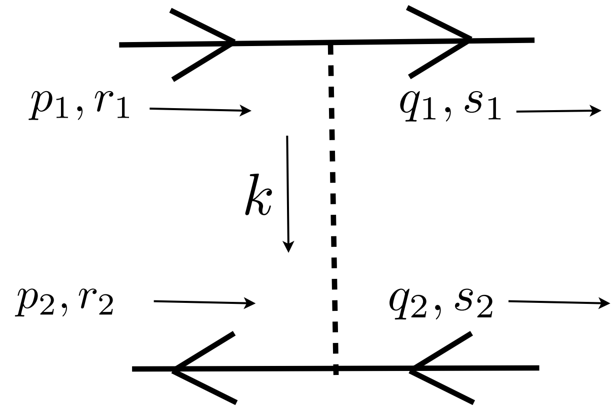
$$\int \frac{d^4k}{(2\pi)^4}\frac{i(\gamma^\mu k_\mu + M)}{k^2 - M^2 + i\epsilon}$$

- Nucleon propagator is now a 4x4 matrix
- Spinor indices are contracted at each vertex
- Add minus signs for statistics

Nucleon-Anti-Nucleon Scattering

$\psi\bar{\psi} \to \psi\bar{\psi}$





- Define amplitude by $\langle f|S-1|i\rangle = i\mathcal{A}(2\pi)^4\delta^4(k_F - k_I)$

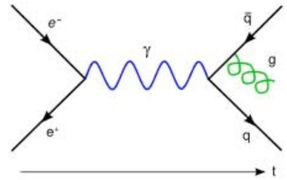
$$\mathcal{A} = (-ig)^2\left[\frac{[\bar{v}^{r_2}(\mathbf{p}_2)\cdot u^{r_1}(\mathbf{p}_1)][\bar{u}^{s_2}(\mathbf{q}_2)\cdot v^{s_1}(\mathbf{q}_1)]}{s-m^2+i\epsilon} - \frac{[\bar{u}^{s_1}(\mathbf{q}_1)\cdot u^{r_1}(\mathbf{p}_1)][\bar{v}^{r_2}(\mathbf{p}_2)\cdot v^{s_2}(\mathbf{q}_2)]}{t-m^2+\cancel{i\epsilon}}\right]$$

$$t = (p_1-q_1)^2 = (p_2-q_2)^2 \qquad u = (p_1-q_2)^2 = (p_2-q_1)^2$$

$$s = (p_1+p_2)^2 = (q_1+q_2)^2$$

- If $m > 2M$ the s-channel term can again diverge. However, the meson is unstable for this mass

Nucleon-Anti-Nucleon Scattering



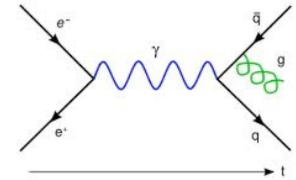
- The minus signs can be a little tricky to get right
- Safest thing to do is to go back to the calculation of the S-matrix element using Wick's theorem
- For the s-channel of nucleon-anti-nucleon scattering this is given (very) schematically by

$$\langle f| : \bar{\psi}\,\psi\,\bar{\psi}\,\psi : |i\rangle = \langle 0|cb : \bar{v}c\,ub\,\bar{u}b^\dagger\,vc^\dagger : b^\dagger c^\dagger|0\rangle$$
$$= +\langle 0|cbb^\dagger c^\dagger[\bar{v}u][\bar{u}v]cbb^\dagger c^\dagger|0\rangle = +[\bar{v}u][\bar{u}v]$$

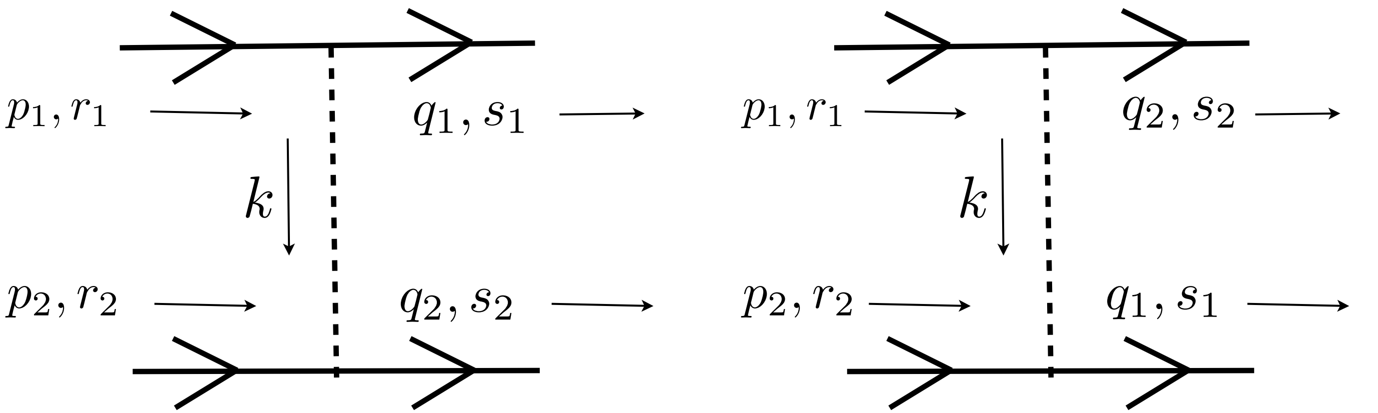
- The t-channel term is

$$\langle f| : \bar{\psi}\,\psi\,\bar{\psi}\,\psi : |i\rangle = \langle 0|cb : \bar{v}c\,vc^\dagger\,\bar{u}b^\dagger\,ub : b^\dagger c^\dagger|0\rangle$$
$$= +\langle 0|cbc^\dagger b^\dagger[\bar{v}v][\bar{u}u]cbb^\dagger c^\dagger|0\rangle = -[\bar{v}v][\bar{u}u]$$

Nucleon Scattering



$$\psi\psi \to \psi\psi$$

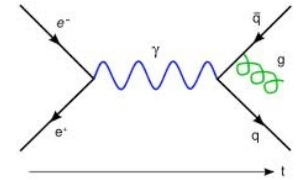


$$\mathcal{A} = (-ig)^2 \left[\frac{[\bar{u}^{s_1}(\mathbf{q}_1) \cdot u^{r_1}(\mathbf{p}_1)][\bar{u}^{s_2}(\mathbf{q}_2) \cdot u^{r_2}(\mathbf{p}_2)]}{t - m^2 + \cancel{i\epsilon}} - \frac{[\bar{u}^{s_2}(\mathbf{q}_2) \cdot u^{r_1}(\mathbf{p}_1)][\bar{u}^{s_1}(\mathbf{q}_1) \cdot u^{r_2}(\mathbf{p}_2)]}{u - m^2 + \cancel{i\epsilon}} \right]$$

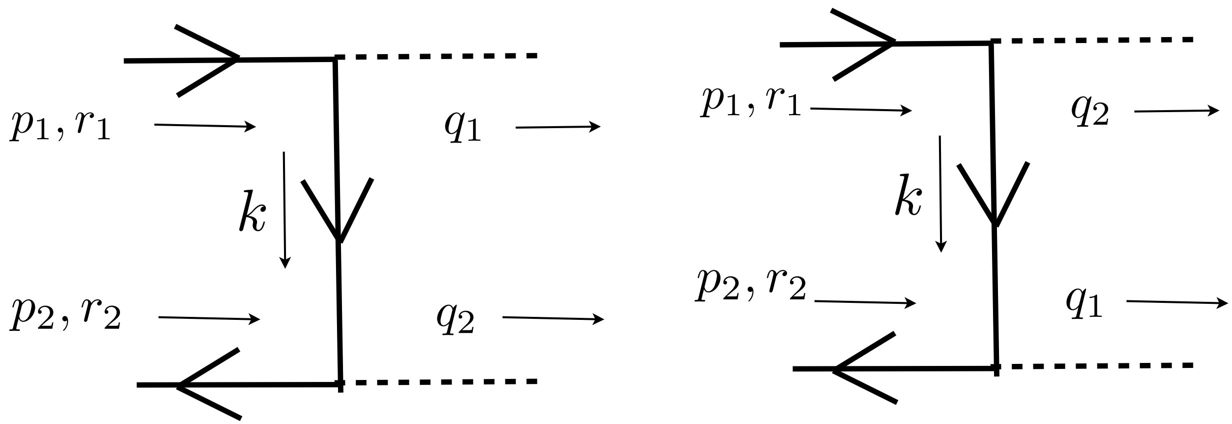
- Notice relative minus sign

(cf. scalar Yukawa theory result in lecture 6)

Nucleon-Meson Scattering



$\psi\bar{\psi} \to \phi\phi$

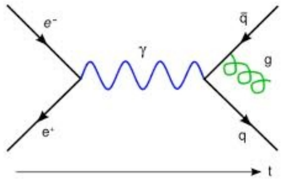


$$\mathcal{A} = (-ig)^2 \left[\frac{\bar{v}^{r_2}(\mathbf{p}_2)\left[\gamma \cdot (p_1 - q_1) + M\right] u^{r_1}(\mathbf{p}_1)}{t - M^2 + \cancel{i\epsilon}} + \frac{\bar{v}^{r_2}(\mathbf{p}_2)\left[\gamma \cdot (p_1 - q_2) + M\right] u^{r_1}(\mathbf{p}_1)}{u - M^2 + \cancel{i\epsilon}} \right]$$

- Here exchange particle is nucleon rather than meson
- Final states are mesons: no relative minus sign

(cf. scalar Yukawa theory result in lecture 6)

The photon field and QED



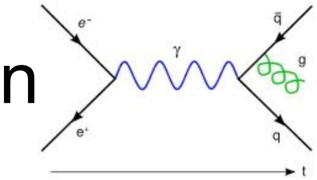
- In this course we have not quantised the vector field A_μ whose excitations are photons (quantisation proceeds in a similar way, but there are new subtleties)

- Its coupling to matter is determined by symmetry: demand that the global U(1) symmetry of our fermion Lagrangian survives for spacetime-dependent parameter

→ Find that this can be maintained if we change our partial derivative to a covariant derivative, linear in a vector field that is subject to gauge transformations

- Studied in the QED course

Glimpse at vector field quantisation



- Canonical quantisation of vector field is subtle due to gauge invariance (easier to perform functional quantisation)

- Consider Maxwell equations $\partial_\mu F^{\mu\nu} = 0$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad \partial_{[\lambda} F_{\mu\nu]} = 0$$

  and note:

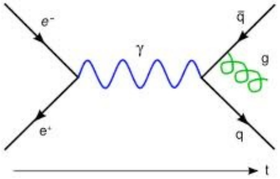
  - No time derivative in A_0
  - Gauge invariance $A_\mu(x) \to A_\mu(x) + \partial_\mu \lambda(x)$

- Expect 4-2 degrees of freedom (2 polarisations for photons)

- However, it's quite subtle to obtain in quantum theory:
  - In Coulomb gauge, subtlety is in modified Poisson brackets due to constraint. D.o.f. manifest but no Lorentz invariance.

  - Let's have a quick look at quantisation in Lorentz gauge. Subtlety is in how to impose the gauge to identify d.o.f.

Glimpse at vector field quantisation



- Work in Lorentz gauge $\partial_\mu A^\mu = 0$

  were e.o.m. reads $\partial_\mu \partial^\mu A^\nu = 0$

- Cheat by modifying action as:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} (\partial_\mu A^\mu)^2$$

  giving the above e.o.m.

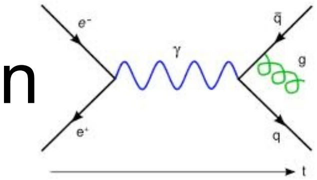
  (Note wrong sign kinetic term for A^0)

- Quantise and impose gauge condition later

- This leads to 4 polarisations $\varepsilon^\lambda(\mathbf{p})$ that can be chosen such that: $\varepsilon^\kappa \cdot \varepsilon^\lambda = \eta^{\kappa\lambda}$

  (1 timelike + 3 spacelike)

Glimpse at vector field quantisation



- For momentum $p \propto (1,0,0,1)$ choose:

$$\varepsilon^0 = (1,0,0,0)^T \qquad \varepsilon^2 = (0,0,1,0)^T$$
$$\varepsilon^1 = (0,1,0,0)^T \qquad \varepsilon^3 = (0,0,0,1)^T$$

  Physical polarisations are ε^1,ε^2. The others must somehow decouple.

- In fact, there is a serious problem with the timelike polarisation ε^0

  Commutation relations

$$[a^\kappa(\mathbf{p}), a^{\lambda\dagger}(\mathbf{q})] = -\eta^{\kappa\lambda}\,(2\pi)^3\,\delta^{(3)}(\mathbf{p}-\mathbf{q})$$

  Notice - sign for $\kappa = \lambda = 0$, so the state

  $a^{0\dagger}(\mathbf{q})|0\rangle$ has negative norm!

- At this point impose Lorentz gauge condition.

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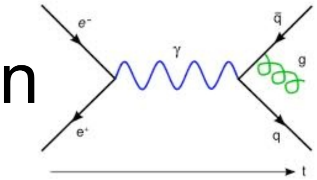
Glimpse at vector field quantisation



But how to impose Lorentz gauge? It appears problematic:

- Cannot impose as an operator equation $\partial_\mu A^\mu = 0$
  (It's too strong: it violates the commutation relation for A^0, π^0)

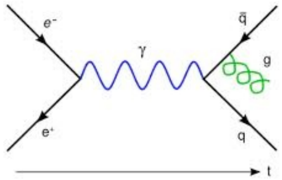
- Cannot impose on physical states as $\partial_\mu A^\mu |\Psi\rangle = 0$
  (Too strong: not even satisfied by the vacuum state!)

- Solution (Gupta-Bleuler) is to impose weaker condition on physical states: $\partial^\mu A^+_\mu |\Psi\rangle = 0$
  (+ sign refers to the +ve freq part of A_μ)

  Implies $(a^3(\mathbf{p}) - a^0(\mathbf{p}))|\phi\rangle = 0$

- The unwanted timelike and longitudinal states combine into a null state of zero norm that decouples from the theory!

Take home message



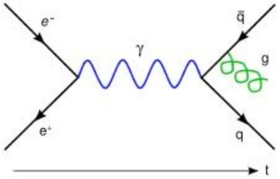
- Developed picture in which particles arise naturally from perturbing quantum fields.

- QFT is not a theory, but a framework for constructing theories (that are local, causal and Lorentz invariant)

- However, it does have a handful of generic predictions:

- There are 2 types of particles: bosons and fermions
- All particles have their anti-particle
- Couplings run with energy scale (renormalisation, not covered in this course)
- Forces are of the Yukawa/Coulomb type (take non-relativistic limit of propagator and interpret as a potential in non-rel QM)

Outlook



In this course we have not covered:

- Quantization of vector field & QED (see QED course)
- Path Integral Formulation of QFT
- Renormalization
- Non-abelian Gauge Theories

Related courses: QED & the Standard Model
Higgs Boson Physics