Asymptotically similar unit root tests in the presence of autocorrelated errors*

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Abstract

The distribution of unit root test statistics generally contain nuisance parameters that correspond to the correlation structure of the innovation errors under the null and alternative hypothesis. The presence of such nuisance parameters can lead to serious size distortions. To address this issue, we adopt an approach based on the characterization of the class of asymptotically similar critical regions for the unit root hypothesis and the application of two new optimality criteria for the choice of a test within this class. This method is designed to address the issue of size stability right from the point of selecting a test. Related methods of Forchini and Marsh (2000) are extended to the case where the innovation sequence takes the form of a moving average process, the order of which is determined by an appropriate information criterion. Limit distribution theory for the resulting test statistics is developed and simulation evidence suggests that our statistics have substantially reduced size distortion while retaining good power properties.

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1 Introduction

The unit root hypothesis has attracted a great deal of interest in econometrics. Nelson and Plosser (1982) provided empirical evidence that many macroeconomic series have a unit root. From the statistical point of view it is important to know whether or not series are stationary in order to conduct valid inference. The outcome of non-stationarity introduces the possibility of differencing the series (Plosser and Schwert, 1978) cointegration (Johansen, 1988) or error-correction (Engle and Granger, 1987) models. Banerjee *et al.* (1993) and Maddala and Kim (1998) give a review of the literature for unit root tests. Fuller (1976) and Dickey and Fuller (1979, 1981) proposed a unit root test (DF) which is widely used.

As in many testing problems, the fact that the distribution of unit root test statistics under the null hypothesis depends on nuisance parameters can result in serious size distortions for the associated unit root tests. Said and Dickey (1984) showed that the "augmented" DF (ADF) test is suitable for processes with autoregressive moving average (ARMA) errors. Phillips and Perron (1988) proposed a nonparametric testing procedure (PP) which allowed for a wider class of stationary time series in the error term. Schwert (1989) used Monte Carlo simulations to show the existence of size distortions in the ADF and PP tests. His results suggest that PP has higher power than ADF, but also much higher size distortions in the presence of negative moving average (MA) parameter in the error term. DeJong *et al.* (1992) showed that PP tests perform poorly against trend stationary alternatives and suggested the use of the Said-Dickey testing procedure.

Ng and Perron (2001) derived a class of unit root tests that take into account possible autocorrelation in the error term. The local asymptotic power function of these tests is close to the Gaussian local power envelope. They also derive the modified information criteria for the choice of the truncation lag. Their simulation study suggest that, for the sample sizes considered, size distortion is very low even in the presence of negative autocorrelation in the innovation sequence. These statistics are described in detail in section 6. Seo (2006) pointed out that a problem exists regarding the global power of these statistics: in finite samples and for alternatives far from the null, the possibility of *power reversal* occurs. Power reversal in this context means that as the true value of the parameter of interest moves farther away from the null hypothesis, power decreases. This problem is caused by the fact that the null of non-stationarity is imposed in the procedure in the construction of the modified information criteria. This type of information criteria could provide very good results with respect to control over size, but it could also have serious flaws when the parameter of interest moves far from the null.

Seo (2006) suggested the use of a two-step procedure in which he firstly fits an autoregression to get the estimated residuals and at the second step uses them as a proxy for the MA component. Perron and Qu (2007) address the issue of power reversal and improve the performance of the statistics by introducing a two step procedure, using OLS estimation for the choice of the order of the lagged differenced terms and GLS estimation for the calculation of the statistics. As can be seen from their results (Figures 1-4), the problem becomes less severe, but is still evident for the case of no autocorrelation in the error term.

This paper addresses the issue of unit root testing in the presence of correlated innovation errors that take the form of a finite order moving average process. Following Hillier (1987), our approach is based on obtaining a characterization of the class of *similar tests*. These are tests whose size does not depend on nuisance parameters, provided that sufficient statistics for the nuisance parameters exist, under the null hypothesis. Given the fact that a sufficient statistic for the MA parameters is not available, we consistently estimate the MA parameters by maximum likelihood, and then use the above estimates to characterize the class of (asymptotically) similar tests. After the characterization of the class of similar regions we proceed to the selection of some tests within this class by the use of appropriate optimality criteria. The advantage of such an approach is that we can focus our attention on a set of tests whose asymptotic size is independent of the nuisance parameters involved. In this way we can address the serious issue of size stability at the first stage of selecting a test.

In order to choose statistics from the class of asymptotic similar tests we make use of the optimality criteria proposed by Forchini and Marsh (2000). They derive unit root tests according to the Bounded Norm Minimizing (BNM) and Bounded Estimated Point Optimal (BEPO) criteria under the assumption of i.i.d. innovation errors. We apply the same optimality criteria to derive tests statistics in a more general framework that allows the presence of possibly correlated innovation errors that may take the form of a finite order MA process. The objective is to derive unit root tests with fairly stable size over MA processes with varying order and values of associated parameters, and with high global power in comparison to other unit root tests existing in the literature.

The paper is organized as follows. In Section 2 we refer to the theory related to the construction of similar tests. Section 3 describes the BNM and BEPO optimality criteria for the choice test statistics proposed by Forchini and Marsh (2000). Section 4 describes the construction of similar regions in the case of correlated errors and in Section 5 we use the optimality criteria to derive the test statistics followed by the description of the method of estimation we are using. The limiting distributions of the resulting test statistics are derived in the presence of deterministics consisting of an intercept term only and an intercept and a linear trend. In Section 6 the finite-sample performance of the statistics is assessed in the context of a simulation study. In Section 7 we provide some concluding remarks. All proofs are included in the technical Appendix of Section 8. Tables and figures are presented in the last section of the paper.

2 Methodology on the characterization of similar regions

The methodology we follow for the characterization of similar regions is described by Hillier (1987). Let z be a vector of random variables with density $f(z; \eta, \theta)$ depending on two vectors of parameters η, and θ. If we want to test the null hypothesis

$$H_0 : \theta = \theta_0$$

then θ is the vector of parameters of interest and η is the vector of nuisance parameters. In general the size of any critical region ω in this context will be dependent on η; ie.,

$$\int_\omega f(z; \eta, \theta_0) = \alpha(\eta).$$

Critical regions related to this problem which are independent of nuisance parameters

$$\int_\omega f(z; \eta, \theta_0) = \alpha$$

are called *similar* critical regions. If there is a sufficient statistic t for η under H_0 the density function is given by

$$f(z; \eta, \theta_0) = pdf(t; \eta, \theta_0) pdf(z|t; \theta_0)$$

where $pdf(t; \eta, \theta_0)$ is the density of the sufficient statistic under H_0 and $pdf(z|t; \theta_0)$ is the conditional density of z given t, which is independent of the nuisance parameter η. So, provided we have sufficient statistics for η, the conditional distribution of z given these statistics will be free of nuisance parameters and will result in a similar critical region.

3 Optimality criteria

We now address the question of how to select a particular test from within the class of similar tests. Ideally, we would choose a *Uniformly Most Powerful*

(UMP) test. A UMP test is a test which has the highest available power for every η, and θ. In unit root tests the power of a test depends on the nuisance parameters η and the value of the parameter of interest θ under H_1, so it is not possible to achieve the UMP criterion. Consequently, we have to use weaker optimality criteria for the selection of test. Cox and Hinkley (1974) suggest some alternative optimality criteria, such as the selection of a typical alternative for θ (*point optimal* (PO)) or the construction of a *locally most powerful* (LMP) test, which involves the maximization of the power of the test in the neighborhood of the null hypothesis. Selecting a typical value of θ could be seen as arbitrary unless there is specific prior information for the parameter. The problem with the LMP tests is that their power often can be low for alternatives that lie far from the null (Zaman, 1996, pp. 133-136).

Forchini and Marsh (2000) suggest the use of two alternative optimality criteria. Their statistical framework can be summarized as follows. Consider a $N \times 1$ vector of observables and a vector of unknown parameters $(\theta, \sigma^2) \in \mathbb{R} \times (0, \infty)$. The null hypothesis $H_0 : y \sim N(0, \sigma^2\Omega(\theta_0))$ is tested against $H_1 : y \sim N(0, \sigma^2\Omega(\theta))$ using the critical region

$$\frac{y'\Omega^{-1}(\theta)y}{y'\Omega^{-1}(\theta_0)y} < k_\alpha \tag{1}$$

where k_α is chosen so that α is the size of the test. It is clear that when the numerator changes with θ there is no a UMP test.

In the absence of a UMP test two weaker optimality criteria are presented in sections 3.1 and 3.2 below.

3.1 Bounded Norm Minimizing tests

Suppose that $y'\Omega^{-1}(\theta)y \leq l(\theta)'\Psi(y)l(\theta)$, where $l(\theta)$ is a vector depending only upon θ and $\Psi(y)$ is a positive definite matrix depending only upon y.

A sufficient condition for

$$\frac{l(\theta)'\Psi(y)l(\theta)}{y'\Omega^{-1}(\theta_0)y} < k_\alpha$$

is to minimize the norm

$$\left\| \frac{\Psi(y)}{y'\Omega^{-1}(\theta_0)y} \right\| < k,$$

for k such that the size of the test is α. The norm in the above equation can be any matrix norm (see e.g. Horn and Johnson, 1985). Notice that any norm of the matrix $\Psi(y)/y'\Omega^{-1}(\theta_0)y$ gives a *norm minimizing* (NM) test and when (1) holds with equality and a BNM test when the inequality is strict.

3.2 Bounded Estimated Point Optimal Tests

The second optimality criterion is that of using *estimated point optimal* tests (EPO). This criterion is related to the PO tests which are discussed above. Even if the alternative is generally unknown, it is possible to estimate it with the value θ^* which satisfies

$$\theta^* = \arg\min_{\theta} \left\{ \frac{l(\theta)'\Psi(y)l(\theta)}{y'\Omega^{-1}(\theta_0)y} \right\}$$

for a set of observations y. In the case where (1) holds with equality, the EPO critical region is given by

$$\frac{l(\theta^*)'\Psi(y)l(\theta^*)}{y'\Omega^{-1}(\theta_0)y} < k, \tag{2}$$

where k is chosen such that the size of the test is α. As with the case of the BNM criterion, if (1) does not hold with equality, (2) is a BEPO test. Another criterion of this type is to reject H_0 if

$$|\theta^* - \theta_0| > k_a, \tag{3}$$

where θ_0 is the value of the parameter under H_0 and k_a is chosen such that the size of the test is a.

Forchini and Marsh (2000) use the above criteria for the derivation of similar unit root test statistics. Simulation results suggest that these statistics have distorted size in the presence of an MA(1) error. In the presence of an MA process in the errors, these test statistics are no longer similar due to the fact that their critical regions depend on the associated MA parameters. The approach in this paper is to modify the construction of the UMP critical region in order to take into account the possibility of an MA(m) process in the errors. Then we apply the BNM and BEPO optimality criteria to choose statistics from the class of asymptotically similar tests and we find that these have good power properties in finite samples.

4 Construction of similar critical regions

Marsh (2005) considers a linear regression model with an MA term in the errors and characterizes the class of asymptotically similar tests. We use the BNM and BEPO optimality criteria for deriving tests within this class. The model is

$$y = X\beta + u, \tag{4}$$

where β is a $k \times 1$ vector of parameters. X is an $N \times k$ full rank matrix of the deterministic components which can either be an intercept, or an intercept and a trend, $u = (u_1, ..., u_N)'$ and

$$\begin{aligned} u_t &= \rho u_{t-1} + \zeta_t \\ \zeta_t &= \sum_{j=0}^{m} \phi_j \varepsilon_{t-j} \\ \varepsilon_t &\sim NIID(0, \sigma^2) \end{aligned}$$

for $t = 1, ..., N$, $u_0 = 0$, and $\phi_0 = 1$. We impose the invertibility condition $\left|\phi_j\right| < 1$ for $j = 1, ..., m$. So the parameters involved are $\theta = (\rho, \beta', \sigma^2, \phi')$ with parameter space $\Theta = (-1, 1] \times \mathbb{R}^k \times \mathbb{R}^+ \times (-1, 1)^m$.

In the context of (4) the unit root hypothesis takes the form

$$H_0 : \rho = 1 \text{ vs. } H_1 : |\rho| < 1,$$

with β, σ^2 and ϕ the nuisance parameters for this testing problem. The method described in section (2) is going to be applied for the construction of similar critical region for the hypothesis stated above. Invariant transformations are applied on the data y, which do not affect the decision with respect to H_0 and H_1, but take out the effect of the nuisance parameters. These transformations involve the use of some matrices defined below.

Let $L^{(i)}$ be the lower-triangular matrix with ones on the i^{th} off-diagonal and zeros elsewhere. Multiplying (from any side) $L^{(i)}$ by any vector gives the i^{th} lag of this vector leaving the first element of the vector unchanged. For this reason we refer to $L^{(i)}$ as the lag-matrix. Using $L^{(i)}$, T_ρ is defined as

$$T_\rho = (I_N - \rho L^{(1)}). \tag{5}$$

Notice therefore that $T_1 = I_N - L^{(1)}$. Multiplying any vector by T_1 results the vector of first differences for the last $N-1$ elements leaving the first element unchanged (implicitly a zero initial condition is imposed). So T_1 acts as a first difference operator that transforms an $I(1)$ series to $I(0)$ except from the the first element which remains unchanged and is asymptotically negligible.

Then, using the $L^{(i)}$ matrix again K_ϕ is defined as

$$K_\phi = (I_N + \sum_{i=1}^{m} \phi_i L^{(i)}). \tag{6}$$

So when the K_ϕ matrix is multiplied by a vector of white noise errors this results in a MA vector series of order m. Using this rationale K_ϕ^{-1} transforms a vector of MA(m) to a vector of white noise series. Defining

$$\phi = (\phi_1, ..., \phi_m)' \text{ and } \varepsilon = (\varepsilon_1, \varepsilon_2, ..., \varepsilon_N)',$$

(4) can be expressed as

$$T_\rho(y - X\beta) = K_\phi \varepsilon. \tag{7}$$

At this point, the transformation matrices listed above are used to clear the distribution of the vector of observables from the nuisance parameters. We start from the joint sample density of y, which is

$$y \sim N(X\beta, \sigma^2 T_\rho K_\phi K'_\phi \left(T_\rho^{-1}\right)').$$

Then, for notational simplicity, we define

$$x = K_\phi^{-1} T_1 y, \tag{8}$$

$$Z = K_\phi^{-1} T_1 X, \tag{9}$$

and (7) is transformed under H_0 to

$$x = Z\beta + \varepsilon.$$

The distribution of x is given by

$$x \sim N(Z\beta, \sigma^2 \Sigma_{\rho,\phi})$$

where

$$\Sigma_{\rho,\phi} = K_\phi^{-1} T_1 T_\rho^{-1} K_\phi K'_\phi \left(T_\rho^{-1}\right)' T'_1 \left(K_\phi^{-1}\right)'. \tag{10}$$

Note that x under H_0 is

$$x \sim N(Z\beta, \sigma^2 I_N)$$

At this point it is useful to use the following lemma before proceeding.

Lemma 1. *The matrix $\Sigma_{\rho,\phi}$ given in (10) can be expressed as*

$$\Sigma_{\rho,\phi} \equiv \Sigma_\rho = T_1 T_\rho^{-1} \left(T_\rho^{-1}\right)' T'_1.$$

For the characterization of the class of similar tests the methodology by Hillier (1987) described in section 2 is applied in this setup. Using the Cholesky decomposition, the projection matrix

$$M_Z = I_N - Z\left(Z'Z\right)^{-1} Z'$$

can be decomposed as:

$$\begin{aligned} CC' &= M_Z \\ C'C &= I_{N-k} \end{aligned}$$

where C is a $N \times N-k$ matrix.

The following transformation are applied using C matrix. First x is transformed as

$$x \longmapsto \begin{pmatrix} \widehat{\beta} = (Z'Z)^{-1} Z'x \\ w = C'x \end{pmatrix}$$

and then w as

$$w \longmapsto \begin{pmatrix} s^2 = \|w\|^2 = x'M_Z x \\ \upsilon = \frac{w}{\|w\|} = C'x/s \end{pmatrix}$$

As it can be seen from the above, $\widehat{\beta}$ is not feasible due to the fact that is dependent on ϕ. It is possible however to proceed by finding a consistent estimate of ϕ.

The distribution of w is

$$w \sim N(0, \sigma^2 C'\Sigma_\rho C) \overset{H_0}{\sim} N(0, \sigma^2 I_{N-k}) \tag{11}$$

Marsh (2007) gives the density of υ with respect to the normalized Haar measure on the surface of the unit $N-k$ sphere to be

$$pdf(\upsilon) = \det \left(C'\Sigma_\rho C\right)^{-1/2} \left[\upsilon' \left(C'\Sigma_\rho C\right)^{-1} \upsilon\right]^{-\frac{N-k}{2}}, \tag{12}$$

According to the above, the most powerful critical region of H_0 vs. H_1 has critical region given by

$$\upsilon' \left(C'\Sigma_\rho C\right)^{-1} \upsilon < k_\alpha, \tag{13}$$

where k_α is chosen such that the size of the test is α.

5 Asymptotically similar statistics

After the characterization of the class of asymptotically similar statistics we use the optimality criteria suggested by Forchini and Marsh (2000) in order to derive test statistics from this class. Since there is not a sufficient statistic for the MA parameters included in matrix K_ϕ, these parameters are estimated using maximum likelihood estimation (MLE). The matrix K_ϕ including the estimated MA parameters is denoted as $K_{\widehat{\phi}}$. More explicitly we define

$$Z_{\widehat{\phi}} = K_{\widehat{\phi}}^{-1} T_1 X \text{ and } \nu = M_{Z_{\widehat{\phi}}} K_{\widehat{\phi}}^{-1} T_1 y. \tag{14}$$

The procedure that gives the order of the MA process and the estimation of the MA parameters is described in detail later in section 5.1. We define

$$\Psi_{11} = \left(T_1^{-1}\right)' T_1^{-1} \tag{15}$$

$$\Psi_{12} = \left(T_1^{-1}\right)' \left(T_1^{-1} - I_N\right) \tag{16}$$

$$\Psi_{22} = \left(T_1^{-1} - I_N\right)' \left(T_1^{-1} - I_N\right). \tag{17}$$

Theorem 2. *Let $\|.\|$ denote a norm on the space 2×2 positive definite matrices, and let*

$$\Psi(\nu) = \frac{1}{\nu'\nu} \begin{pmatrix} \nu'\Psi_{11}\nu & \nu'\Psi_{12}\nu \\ \nu'\Psi_{12}\nu & \nu'\Psi_{22}\nu \end{pmatrix} \tag{18}$$

Then a BNM test is: reject $H_0 : \rho = 1$ if

$$N^{-1} \|\Psi(\nu)\| < k_\alpha \tag{19}$$

where ν is defined in (14), and k_α is chosen such that the size of the test is α.

Theorem 2 generates a class of BNM tests, depending upon the choice of particular norm. A statistic from this class could result from the use of the Euclidean matrix norm $\|\Psi(\nu)\| = \{tr\Psi(\nu)'\Psi(\nu)\}^{1/2}$ or the spectral norm of $\Psi(\nu)$, defined as the square root of the maximal eigenvalue of $\Psi(\nu)'\Psi(\nu)$. For the statistics derived in this paper $\|\Psi(\nu)\| = 2$.

Theorem 3. *A BEPO test for $H_0 : \rho = 1$ against $H_1 : -1 < \rho < 1$ is given by the following rule:*
reject H_0 if

$$BEPO = N \left| \frac{\nu'\Psi_{12}\nu - \nu'\Psi_{22}\nu}{\nu'\Psi_{22}\nu} \right| > k_\alpha \tag{20}$$

where ν is defined in (14) and k_a is such that the size of the tests is α.

5.1 Estimation of the MA process

Both the BNM and BEPO statistics contain the matrix $K_{\widehat{\phi}}$ of estimated MA coefficients. The construction of this matrix requires two steps: a procedure that detects the order of the MA component and an estimation method for the MA parameters. Treating both these aspects as a priori unknown makes the inference of Theorems 2 and 3 asymptotically feasible and suitable for practical application.

We first discuss the estimation of the MA parameters for a given order. In the absence of a sufficient statistic for ϕ, we need to employ a consistent estimator. It has to be stressed that the choice of a good estimator for ϕ is of major importance for the good properties (empirical size near to the nominal one and high power) of the statistics. We estimate ϕ by conditional maximum likelihood or pseudo-maximum likelihood if we do not wish to maintain the normality assumption on the innovation errors. It is a well known fact that, under the invertibility assumption imposed on the moving average process, the (pseudo) maximum likelihood estimator of ϕ is $\sqrt{N}$-consistent.

Having estimated models of certain order m, we use information based rules to choose one among them. These are the criteria proposed by Akaike (1974), Schwarz (1978) and Hannan and Quinn (1979), denoted henceforth as AIC, BIC and HQIC respectively. These are described in detail below.

The algorithm for estimating ϕ is described below. We first estimate the following model with least squares:

$$y_t = X\widehat{\beta} + \hat{u}_t, \tag{21}$$

where X includes an intercept only, or an intercept and a trend. We then fit the following $ARMA(1, m)$ model on the residuals of (21)

$$\widehat{u}_t = \rho\widehat{u}_{t-1} + \varepsilon_t + \sum_{i=1}^{m} \phi_i \varepsilon_{t-i},$$

for $t = 1, 2, ..., N$. We set a minimum value $m_{\min}$, and a maximum value $m_{\max}$ for the order of the MA component. We estimate $ARMA(1, m)$ models with $m_{\min} \leq m \leq m_{\max}$. For each model, we condition on the m first values of ε being zero:

$$\varepsilon_0 = \varepsilon_1 = ... = \varepsilon_m = 0.$$

From the above assumptions we can iterate on:

$$\varepsilon_t = (\widehat{u}_t - \rho\widehat{u}_{t-1}) - \sum_{i=1}^{m} \phi_i \varepsilon_{t-i},$$

for $t = 1, 2, ..., N$.

The conditional log likelihood is

$$\mathcal{L}\left(\rho, \phi, \sigma^2\right) = -\frac{N}{2}\log(2\pi) - \frac{N}{2}\log(\sigma^2) - \sum_{t=1}^{N} \frac{\varepsilon_t^2}{2\sigma^2}$$

Since we assumed $\left|\phi_j\right| < 1$ for $j = 1, ..., m$ the effect of the initial condition fades out as sample size increases (Hamilton p.128).

After the estimation of $m_{\max} - m_{\min} + 1$ models we use information criteria to choose one of them. These information criteria are the following

$$\begin{aligned} IC_{AIC}(m) &= -2\frac{\mathcal{L}}{N} + \frac{2(m+1)}{N} \\ IC_{BIC}(m) &= -2\frac{\mathcal{L}}{N} + \frac{(m+1)\ln(N)}{N} \\ IC_{HQIC}(m) &= -2\frac{\mathcal{L}}{N} + \frac{2(m+1)\ln(\ln(N))}{N}. \end{aligned}$$

We choose m such that the information criterion (used in each case) is minimized:

$$\widehat{m} = \arg\min_{m} IC(m).$$

After choosing the order of the MA component and estimating the MA parameters, we can substitute them in the sufficient statistics for (β, σ^2) and then construct the similar critical regions. It is important to note here that, asymptotically, the test statistics we derive do not depend on the nuisance parameter under H_0 since $Z = K_{\hat{\phi}}^{-1} T_1 X$ and $T_1 u = K_{\phi} \varepsilon$ which gives

$$\begin{aligned} \nu &= M_Z K_{\hat{\phi}}^{-1} T_1 (X\beta + u) = M_Z K_{\hat{\phi}}^{-1} T_1 u \\ &= M_Z K_{\hat{\phi}}^{-1} K_{\phi} \varepsilon = [I + o_p(1)] M_Z \varepsilon. \end{aligned}$$

The above result shows that the statistics we derive are asymptotically similar.

5.2 Limiting distribution of BNM and BEPO statistics

Having derived the BNM and BEPO test statistics for the unit root hypothesis, we proceed to derive their limiting distributions. To this end, we restrict the deterministic components of the data generating process to an intercept and a linear trend, i.e. we assume that the matrix of deterministics in (4) takes the form

$$X' = \begin{bmatrix} 1 & 1 & ... & 1 \\ 1 & 2 & ... & N \end{bmatrix}, \tag{22}$$

or

$$X' = \begin{bmatrix} 1 & 1 & ... & 1 \end{bmatrix}, \tag{23}$$

which corresponds to the case where only an intercept is included in the model.

Theorem 4. *Consider the process in (4) and let $W(\cdot)$ be standard Brownian motion on $D[0,1]$. Under the null hypothesis $H_0 : \rho = 1$ the following limit theory applies as $N \to \infty$:*

For X satisfying (22)

(i) *The BNM test of Theorem 2 satisfies*

$$BNM \Rightarrow 2\left\{\int_0^1 W^2(r)dr - 2W(1)\int_0^1 rW(r)dr + \frac{1}{3}W^2(1)\right\}.$$

(ii) *The BEPO test of Theorem 3 satisfies*

$$BEPO \Rightarrow \frac{1}{2}\frac{1}{\left|\int_0^1 W^2(r)dr - 2W(1)\int_0^1 rW(r)dr + \frac{1}{3}W^2(1)\right|}.$$

For X satisfying (23):

(iii) *The BNM test of Theorem 2 satisfies*

$$BNM \Rightarrow 2\left\{\int_0^1 W^2(r)dr\right\}.$$

(iv) *The BEPO test of Theorem 3 satisfies*

$$BEPO \Rightarrow \frac{1}{2}\left|\frac{W^2(1)-1}{\int_0^1 W^2(r)dr}\right|.$$

6 Numerical Study

The test statistics we develop are motivated asymptotically in the sense that they are asymptotically similar with respect to the MA parameter. In order to examine their size and power properties in small samples we employ a Monte Carlo study. Two models are considered for the simulations: the first is based on (4) with X defined as in (22) for the case of a constant and trend included and (23) for the case of a constant only included. The DGP used for the simulations has the following specification:

$$\begin{aligned} u_t &= \rho u_{t-1} + \varepsilon_t + \phi\varepsilon_{t-1}, \\ \varepsilon_t &\sim NIID(0,1), \end{aligned}$$

Each Monte Carlo experiment was based on 10000 replications. We investigate size distortion and power of the statistics in finite samples. For the numerical study related to size distortion, the following minimal complete factorial design is used with values for the parameters

$$\begin{aligned} \phi &= -0.8, -0.7, ..., 0.8, \\ N &= 50, 100, 200, 400, \\ \rho &= 1, \\ \alpha &= 0.05, \end{aligned}$$

where α is the nominal size of the test statistics. For the numerical study investigating the finite sample power of the statistics, the simulation design includes all combinations of the following parameter values

$$\begin{aligned} \rho &= 0.8, 0.82, ..., 0.98, \\ N &= 50, 100, 200, 400, \\ \phi &= -0.5, 0, \\ \alpha &= 0.05, \end{aligned}$$

and

$$\begin{aligned} \rho &= 0.1, 0.2, ..., 0.9, \\ N &= 50, 100, 200, 400, \\ \phi &= 0, \\ \alpha &= 0.05. \end{aligned}$$

The statistics BNM_0 and $BEPO_0$ correspond to the case in which MA terms are not estimated. These are the statistics proposed by Forchini and Marsh (2000). In order to get statistics BNM_0 and $BEPO_0$ we set $\phi = 0$ (i.e. $K_\phi = I_N$) in (19) and (20) respectively. For BNM_a and $BEPO_a$ statistics the AIC is used, for BNM_b and $BEPO_b$ the BIC, and for BNM_h and $BEPO_h$ the HQIC. We refer to these test statistics as similar statistics. The information criteria consider MA(m) processes with $m_{\min} = 0$ and $m_{\max} = 5$. Throughout this simulation study we use exact critical values for the statistics resulting from the BNM and $BEPO$ criteria.

We compare the finite sample performance of the statistics derived in this paper with other statistics in the literature. In Ng and Perron (2001) the following statistics can be found:

$$\begin{aligned} MZ_a^{GLS} &= \frac{N^{-1}\tilde{y}_N^2 - s_{AR}^2}{2N^{-2}\sum_{t=1}^{N}\tilde{y}_{t-1}^2}, \\ MSB^{GLS} &= \left(\frac{N^{-2}\sum_{t=1}^{N}\tilde{y}_{t-1}^2}{s_{AR}^2}\right)^{\frac{1}{2}} \\ MZ_t^{GLS} &= MZ_a^{GLS} \times MSB^{GLS}, \end{aligned}$$

where $\tilde{y}_t = y_t - x_t\widehat{\gamma}^{GLS}$, ($x_t$ being the t-th row of X) and $\widehat{\gamma}^{GLS}$ being the GLS estimate of γ. This is calculated by the GLS regression of $y_t^{\overline{a}}$ on $x_t^{\overline{a}}$, where

$y_t^{\overline{a}} = y_t - \overline{a} y_{t-1}^{\overline{a}}$ for $t = 2, ..., N$ and $y_1^{\overline{a}} = y_1$. Following Elliot *et al.* (1996), for the case of a constant only in the model $\overline{a} = 1 + \frac{-7}{N}$, and when a constant and trend are included $\overline{a} = 1 + \frac{-13.5}{N}$.

Ng and Perron also modify the feasible point optimal test suggested Elliot *et al.* (1996) which is

$$P_T = \frac{S(\overline{a}) - \overline{a} S(1)}{s_{AR}^2},$$

where $S(a) = \inf_\gamma \sum_{t=1}^N (y_t^a - \gamma x_t^a)^2$.

The modified point feasible point optimal test suggested by Ng and Perron (2001) for the constant case is

$$MP_T^{GLS} = \frac{\overline{c}^2 N^{-2} \sum_{t=1}^N \tilde{y}_{t-1}^2 - \overline{c} N^{-1} \tilde{y}_N^2}{s_{AR}^2},$$

and for the case of a constant and trend included in the deterministics

$$MP_T^{GLS} = \frac{\overline{c}^2 N^{-2} \sum_{t=1}^N \tilde{y}_{t-1}^2 + (1 - \overline{c}) N^{-1} \tilde{y}_N^2}{s_{AR}^2}.$$

The autoregressive spectral density estimate of σ^2 is defined

$$s_{AR}^2 = \frac{\hat{\sigma}_{ek}^2}{\left(1 - \sum_{t=1}^N \hat{b}_i\right)^2},$$

$$\hat{\sigma}_{ek}^2 = N^{-1} \sum_{t=k+1}^N \hat{e}_{tk}^2,$$

with $\hat{b}_i$ and e_{tk}^2 derived from the following OLS regression

$$\Delta \tilde{y}_t = \hat{b}_0 \tilde{y}_{t-1} + \sum_{i=1}^k \hat{b}_i \Delta \tilde{y}_{t-i} + \hat{e}_{tk}.$$

Note that the above regression is used for the GLS ADF test. More specifically a t-test is run on $H_0 : b_0 = 0$.

The Modified Akaike Information Criterion used for the determination of the autoregressive order k is:

$$MAIC(k) = \ln\left(\hat{\sigma}_k^2\right) + 2\frac{\tau_T(k) + k}{N - k_{\max}},$$

where

$$\tau_T(k) = \frac{\hat{b}_0 \sum_{t=k_{\max}+1}^{N} \tilde{y}_{t-1}^2}{\hat{\sigma}_k^2},$$

$$\hat{\sigma}_k^2 = \frac{\sum_{t=k_{\max}+1}^{N} \hat{e}_{tk}}{N - k_{\max}}.$$

The upper bound is set to $k_{\max} = int\left(12(N/100)^{1/4}\right)$. The value of k chosen by $MAIC(k)$ is the one such that $k = \arg\min_{k\in[0,k_{\max}]}$.

In the tables of this paper,MZ_a and MZ_t are the modified PP statistics and MSB is the modified Sargan-Bhargava statistic. P_T refers to the feasible point optimal test and MP_T to its modified variant. All these statistics use GLS detrending. ADF corresponds to the ADF statistic with GLS detrending and for ADF_{LS} OLS detrending is used. MZ_{aLS} statistic denotes the MZ_a statistic based on OLS detrending. Lastly, MZ_{a2} corresponds to the MZ_a statistic with GLS detrending used for the data and OLS detrending used for the spectral density estimation.

Tables 1a ($N = 50, 100$) and 1b ($N = 200, 400$) report size distortion of the statistics for a model including an intercept term only (X defined by (23)) and 2a ($N = 50, 100$) and 2b ($N = 200, 400$) report the size distortion of the statistics for the case of an intercept and a trend included in the model (X defined by(22)). A first observation is that serious size distortions occur when the MA parameter is specified to be near to -1. These tables show that the statistics derived in this paper exhibit much lower size distortion in comparison to the BNM_0 and $BEPO_0$ statistics. It can be also seen that the choice of the specific information criterion is crucial for the level of size distortion in small samples ($N = 50, 100$). More specifically, the size distortion for our statistics is the lowest when the AIC is used. When the HQIC is used, size distortion becomes higher and the use of BIC gives the highest size distortion among all information criteria considered for our statistics. The relatively good performance of BNM_a and $BEPO_a$ with respect to size distortion could be explained by the fact that the AIC is the most "liberal" (tends to choose comparatively higher order for the MA process) of all information criteria. This is evident in Figure 1 which presents the relative frequencies of the order chosen by each information criterion under H_0 for different values of ϕ (for a model with an intercept and trend, sample size $N = 100$). It can be seen that BIC is the most "conservative" information criterion, in the sense that, keeping everything else constant, it tends to choose the lowest MA order in comparison to the other two criteria. This has a very detrimental effect for values of ϕ close to -1 and that is why BNM_b and $BEPO_b$ give the highest size distortion in small samples. It is also observed that for sample sizes $N = 200, 400$ the different information criteria deliver almost the same empirical size.

Comparing our statistics with other statistics in the literature, we find that they have much lower size distortion for $N = 50$. For values of the MA parameter being close to -1, it is obvious that all the the statistics of Ng and Perron (2001) have extremely high size distortion, making them not reliable for such small sample sizes. This is important since sample sizes of this kind are relevant in applied research. For higher sample sizes, statistics MZ_a, MZ_t, MSB, P_T and MP_T appear to have very small size distortion and perform better than the similar statistics. Figures 2 and 3 illustrate graphically the facts mentioned above.

Another crucial observation for the similar statistics is that their size distortion reduces as the sample size N increases. For example, in the case of a model with intercept only (Tables 1a and 1b), when $\phi = -0.8$, $BEPO_a$ statistic has size 0.367 for $N = 50$, 0.22 for $N = 100$, 0.101 for $N = 200$, and 0.07 for $N = 400$. We observe the same behaviour for BNM_b, $BEPO_b$, BNM_h and $BEPO_h$. This observation suggests that the empirical size of the similar statistics derived in this paper converges to its nominal value (5% in this case), as sample size increases. This can be attributed to the consistency of the maximum likelihood estimator, as well as the better performance of the information criteria as N increases. This is not the case for statistics BNM_0 and $BEPO_0$: size distortion increases as sample size N increases. The $BEPO_0$ statistic for example has size 0.673 for $N = 50$, 0.832 for $N = 100$ and 0.908 for $N = 200$, when $\phi = -0.8$. This suggests that empirical size of the BNM_0 and $BEPO_0$ statistics can go farther from nominal size as N increases in the presence of autocorrelation in the errors. In the case of an intercept and a trend included in the model (Tables 2a and 2b) we observe that the level of size distortion increases for all the statistics.

Tables 3 and 4 report the power of the statistics for models corresponding to X defined by (23) and (22) respectively, when there is no autocorrelation in the error term ε_t ($\phi = 0$). This is not a favourable case for the statistics we derive in this paper, since MA processes are considered which do not exist under the data generating process. However, we observe that the power of BNM_b and $BEPO_b$ statistics is very close to the power of BNM_0 and $BEPO_0$ (which do not assume autocorrelation of ε_t). The BNM_a and $BEPO_a$ statistics have substantially lower power than the other statistics. The power of BNM_h and $BEPO_h$ statistics is lower that the power of BNM_b and $BEPO_b$, but close to it. For sample sizes $N = 200, 400$ the choice of a specific information criterion does not make any substantial difference with respect to the level of power of the statistics.

Table 3 shows that MZ_a, MZ_t and MSB for $N = 50$ have substantially higher power than our statistics. For N higher that 100 our statistics appear to outperform the modified statistics derived by Ng and Perron (2001). The ADF statistic appears to have comparatively high power across all sample sizes considered. In the case of an intercept and trend included in the model (Table 4), we observe that statistics MZ_a, MZ_t, MSB, P_T, MP_T, MZ_{aLS} and MZ_{a2} have extremely low power (smaller than the 5% size for alternatives close to H_0). The ADF and ADF_{LS} statistics appear to have higher power compared to our statistics. For sample sizes higher than 100, BNM_b and $BEPO_b$ statistics have higher power than the ADF statistic.

Table 5 presents the results for size-adjusted power for the model including an intercept only, when there is negative autocorrelation ($\phi = -0.5$) in the error term. A first observation is that the power of our statistics is lower in comparison to the case of no autocorrelation (Table 3) especially for sample sizes 50 and 100. We also observe that the BNM_b, $BEPO_b$, BNM_h and $BEPO_h$ appear to have higher power than BNM_0 and $BEPO_0$ for sample size $N = 50$. For this sample size the modified statistics perform better than our similar statistics. For sample size 100, we observe that BNM_b, $BEPO_b$, BNM_h and $BEPO_h$ have higher power than BNM_0 and $BEPO_0$ and the statistics proposed by Ng and Perron (2001) for alternatives far from the null $\rho = 1$. For alternatives $0.98 \leq \rho \leq 0.90$ we find that the ADF statistic has higher power. For the same alternatives BNM_0 and $BEPO_0$ have higher power. For higher sample sizes our statistics have comparatively higher (in comparison to the Ng and Perron statistics) power close to the null as well.

Table 6 refers to the case of a model including an intercept and a trend in the presence of negative autocorrelation in the error term ($\phi = -0.5$). First of all, for sample size $N = 50$ we observe that all statistics suffer from the problem of very low power. We also observe that for most alternatives, BNM_0 and $BEPO_0$ exhibit higher power than our statistics for sample sizes $N = 50, 100$. Also the statistics derived by the procedure of Ng and Perron (2001) have substantially higher power than ours. For sample sizes higher than $N = 100$ our statistics appear to have higher power for most alternatives.

Tables 7 and 8 contain the finite sample power of the statistics when there is no autocorrelation of the error term, for alternatives farther than the ones investigated in Tables 3 and 4. The reason for this is to examine the possibility of power reversal. Table 7 corresponds to a model with an intercept only. We observe that the problem of power reversal is severe for statistics MZ_a, MZ_t, MSB, P_T, MP_T, MZ_{aLS} and MZ_{a2} for sample sizes

$N = 100, 200$. For example for sample size $N = 100$, alternative 0.8 the power of the MZ_a is 0.839 which is the highest among the values of power computed. Moving away from alternative 0.8 power decreases gradually, reaching power 0.715 for alternative 0.1. ADF and ADF_{LS} statistics do not appear to have this problem. Regarding the statistics derived in this paper we can see that there is such behaviour for BNM_a and $BEPO_a$ statistics. For the same case ($N = 100$) $BEPO_a$ statistic has power 0.958 for $\rho = 0.5$ and then gradually falls to 0.947 for $\rho = 0.1$. We consider the power reversal of $BEPO_a$ to be less serious than the one occurring for MZ_a mainly because of the magnitude of the power reduction: 1.1% for $BEPO_a$ power reduction from alternative 0.5 to 0.1 is 1.1% while the power reduction from alternative 0.8 to 0.1 for MZ_a is 14.8%. Additionally, we observe that our statistics have higher power in comparison to the other test statistics existing in the literature for alternatives far from the null ($\rho = 1$). Figure 4 presents the power of the BNM_b, BNM_a, BNM_h, ADF, and MZ_a for the model including an intercept only. For sample size $N = 50$ the ADF statistic appears to have higher power that the BNM_b, BNM_a and BNM_h statistics. The MZ_a statistic has higher power for alternatives far from the null. For higher sample sizes the BNM_b, BNM_a and BNM_h perform better than ADF and MZ_a. In this figure one can see that the power function of MZ_a changes slope for sample sizes $N = 100, 200$.

The problem of power reversal becomes more apparent in the context of a model which includes an intercept and a trend. This case is presented for the same statistics in Figure 5. In this case one can see that even for a sample size as high as $N = 400$, the MZ_a statistic has a decreasing power as the true value of ρ moves farther away from H_0. Table 8 presents the results for power in the absence of autocorrelation in the errors for all the statistics. For statistics MZ_a, MZ_t, MSB, P_T, MP_T, MZ_{aLS} and MZ_{a2} similar conclusions to the ones of Table 7 can be drawn. Table 8 shows that the problem of power reversal occurs for statistic ADF as well, but not for ADF_{LS}. This problem appears for our statistics being less severe (much smaller power reduction as ρ moves farther away from the null).

Tables 9a ($N = 50, 100$) and 9b ($N = 200, 400$) present the performance of the information criteria across different values of ϕ under H_0, for a model with an intercept only. This could help to explain the difference among our statistics with respect to control over size. As mentioned above, the BIC is the most "conservative" information criterion and AIC is the most "liberal", while HQIC lies in between the other two criteria. As a consequence, statistics that use the AIC have better control over size in the presence of negative MA parameters in comparison to the statistics using the other criteria. For $N = 50$ and value $\phi = -0.8$ under H_0, the BIC chooses order

0 (no autocorrelation) 68.6% of the cases, AIC 39.4% and HQIC 52.7%. As sample size increases the performance of all information criteria is improved (they tend to choose the correct order) and for $N = 400$ and value $\phi = -0.8$, none of the information criteria chooses order 0 (i.e. all criteria suggest that there is autocorrelation in the error term). That is why we do not observe substantial difference with respect to size distortion among our statistics for large samples. Information criteria behave similarly in the case of a model with an intercept and a trend.

Tables 10 and 11 present the performance of information criteria for models including an intercept only and an intercept and a trend respectively, across different values of alternatives in the case of no autocorrelation in the error term ($\phi = 0$). These tables explain the occurrence of the problem of power reversal for some of our statistics. We observe that the AIC performs worse with respect to identifying the right MA order as the true value of ρ moves farther away from the null. In table 10, we see that for $N = 100$ the AIC chooses order 0 (the true under the DGP) for the MA component 70.8% of the cases and for $\rho = 0.1$, 61.5%. For the same sample size, under the null, the BIC chooses order 0 for the MA component 95.6% of the cases and for $\rho = 0.1$, 93.9%. Table 11 shows that moving to a model with an intercept and a trend makes the problem of identifying the right order more serious for the AIC. For this model, and for sample size $N = 100$ the AIC chooses order 0 67.8% of the cases and for $\rho = 0.1$, choice of zero order falls to 55.5%. Under H_0, the BIC chooses order 0 94.8% of the cases and for $\rho = 0.1$, the relative frequency is 92%.

7 Conclusion

In this paper we derive asymptotically similar statistics for testing the unit root hypothesis in the presence of autocorrelated errors. Based on the BNM and BEPO optimality criteria proposed by Forchini and Marsh (2000), we derive test statistics that take into consideration possible autocorrelation in the error term. We consider our testing procedure to be feasible with respect to two aspects. The first involves the use of information criteria (BIC, AIC and HQIC) for the choice of the order of autocorrelation. The second includes the estimation of the parameters of the chosen model. Limiting distributions for the test statistics are provided which enable us to use asymptotic critical values for high sample sizes (over $N = 100$). In order to assess the finite sample performance of our statistics under different specifications, we perform an extensive simulation study.

We believe that we successfully generalize the statistics of Forchini and

Marsh since we improve substantially the size control of the statistics in the presence of autocorrelation, without any significant power loss even in the case of no autocorrelation in the error term.

Additionally, we compare our statistics with a variety of other statistics existing in the literature (mainly the ones in Ng and Perron, 2001). We find that for a small sample size (such as $N = 50$) the other statistics could possibly have so high level of size distortion, that would make inference drawn by them highly unreliable. Our test statistics perform much better with respect to control over size. For higher sample sizes our statistics perform comparatively worse to the Ng and Perron statistics, but size distortion appears to fall substantially as sample size increases. With respect to finite sample power, our statistics achieve higher power for most alternatives apart from the ones close to the null hypothesis. Finally, our statistics do not seem to suffer seriously from the problem of power reversal.

We observe that the optimality criteria used (BNM and BEPO) deliver statistics that have very similar empirical size and power in finite samples. However, what differentiates the finite sample properties of our statistics, is the use of the information criterion for the determination of the order of the MA component. The use of AIC delivers the best results with respect to size control, but also has the lowest power and for some sample sizes the problem of power reversal occurs. The BIC gives the best results with respect to power, but the worse for controlling size in small samples. The HQIC appears to lie in between the other criteria mentioned, delivering test statistics with power close to BNM_b and $BEPO_b$, and size distortion not much higher than the one of BNM_a and $BEPO_a$. We suggest the use of the HQIC, because of the fact that the BNM_h and $BEPO_h$ statistics appear to have comparatively, to the other asymptotically similar statistics, low size distortion, high power and not significant (if any) power reduction for alternatives far from the null.

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8 Technical Appendix and Proofs

Proposition A1. *The lag matrix $L^{(i)}$ commutes with any other lag matrix of different or same order $L^{(j)}$ and*

$$\begin{aligned} K_\phi T_\rho &= T_\rho K_\phi, \\ T_1^{-1} K_\phi^{-1} &= K_\phi^{-1} T_1^{-1}, \\ T_1' K_\phi' &= K_\phi' T_1', \\ \left(T_1^{-1}\right)' \left(K_\phi^{-1}\right)' &= \left(K_\phi^{-1}\right)' \left(T_1^{-1}\right)', \\ K_\phi^{-1} T_\rho &= T_\rho K_\phi^{-1}, \\ \left(K_\phi^{-1}\right)' T_\rho' &= T_\rho' \left(K_\phi^{-1}\right)', \end{aligned}$$

given that K_ϕ and T_ρ are invertible.

Proof. Lag matrix $L^{(i)}$ commutes with any other lag matrix of the same or different order $L^{(j)}$ and:

$$L^{(i)} L^{(j)} = L^{(j)} L^{(i)} = \begin{cases} L^{(i+j)}, & \text{for } i+j \leq N-1 \\ 0, & \text{for } i+j > N-1. \end{cases} \tag{24}$$

Noting the definitions in (5) and (6) and the commutative property of lag matrix $L^{(i)}$ (24) we have:

$$\begin{aligned} K_\phi T_\rho &= \left(I_N + \sum_{i=1}^{q} \phi_i L^{(i)}\right) \left(I_N - \rho L^{(1)}\right) \\ &= I_N - \rho L^{(1)} + \sum_{i=1}^{q} \phi_i L^{(i)} - \left(\sum_{i=1}^{q} \phi_i L^{(i)}\right) \rho L^{(1)} \\ &= I_N - \rho L^{(1)} + \sum_{i=1}^{q} \phi_i L^{(i)} - \rho \sum_{i=1}^{q} \phi_i L^{(i)} L^{(1)} \\ &= I_N - \rho L^{(1)} + \sum_{i=1}^{q} \phi_i L^{(i)} - \rho L^{(1)} \sum_{i=1}^{q} \phi_i L^{(i)} \\ &= I_N - \rho L^{(1)} + \left(I_N - \rho L^{(1)}\right) \sum_{i=1}^{q} \phi_i L^{(i)} \\ &= \left(I_N - \rho L^{(1)}\right) \left(I_N + \sum_{i=1}^{q} \phi_i L^{(i)}\right) = T_\rho K_\phi. \end{aligned} \tag{25}$$

Equation (25) means that K_ϕ commutes with T_ρ (and with T_1 which is a special case of T_ρ). Given that K_ϕ and T_ρ are nonsingular matrices, we can easily show that their respective inverse and transpose matrices commute with each other as well:

$$K_\phi T_\rho = T_\rho K_\phi \Leftrightarrow (K_\phi T_\rho)^{-1} = (T_\rho K_\phi)^{-1} \Leftrightarrow T_\rho^{-1} K_\phi^{-1} = K_\phi^{-1} T_\rho^{-1}, \quad (26)$$

$$K_\phi T_\rho = T_\rho K_\phi \Leftrightarrow (K_\phi T_\rho)' = (T_\rho K_\phi)' \Leftrightarrow T_\rho' K_\phi' = K_\phi' T_\rho', \quad (27)$$

and combining (26) and (27) we get

$$\left(T_1^{-1}\right)' \left(K_\phi^{-1}\right)' = \left(K_\phi^{-1}\right)' \left(T_1^{-1}\right)'. \quad (28)$$

Finally, using (25) we show that T_ρ commutes with K_ϕ^{-1}

$$K_\phi T_\rho = T_\rho K_\phi \Rightarrow T_\rho = K_\phi^{-1} T_\rho K_\phi \Rightarrow T_\rho K_\phi^{-1} = K_\phi^{-1} T_\rho, \quad (29)$$

and transposing both sides of (25) we can show that $\left(K_\phi^{-1}\right)' T_\rho' = T_\rho' \left(K_\phi^{-1}\right)'$.

Proposition A2. *Let $S = T_1^{-1}\varepsilon$ and $\sigma^2 = E\left(\varepsilon_1^2\right)$. Under the assumptions of Theorem 4 with X* satisfying (22), *the following limit theory applies under the null hypothesis $H_0 : \rho = 1$ as $N \to \infty$:*

(i) $N^{-1} S'\varepsilon \Rightarrow \frac{1}{2}\sigma^2 \left[W^2(1) + 1\right]$

(ii) $N^{-1} S' P_Z \varepsilon \Rightarrow \sigma^2 W(1) \int_0^1 W(r)dr$

(iii) $N^{-2} S' T_1^{-1} P_Z \varepsilon \Rightarrow \sigma^2 W(1) \int_0^1 rW(r)dr$

(iv) $N^{-2} \left(T_1^{-1} P_Z \varepsilon\right)' T_1^{-1} P_Z \varepsilon \Rightarrow \frac{1}{3}\sigma^2 W^2(1)$

(v) $N^{-1} \left(T_1^{-1} P_Z \varepsilon\right)' P_Z \varepsilon \Rightarrow \frac{1}{2}\sigma^2 W^2(1)$

(vi) $N^{-1} \left(T_1^{-1} P_Z \varepsilon\right)' \varepsilon \Rightarrow \sigma^2 W(1) \left(W(1) - \int_0^1 W(r)dr\right)$

(vii) $N^{-1} \nu'\nu \to_p \sigma^2$

For X satisfying (23) parts (i) and (vii) continue to apply and:

(viii) $N^{-1} S' P_Z \varepsilon$, $N^{-1} \left(T_1^{-1} P_Z \varepsilon\right)' P_Z \varepsilon$ and $N^{-2} \left(T_1^{-1} P_Z \varepsilon\right)' T_1^{-1} P_Z \varepsilon$ have order $O_p(N^{-1})$ and $N^{-2} S' T_1^{-1} P_Z \varepsilon$, $N^{-1} \left(T_1^{-1} P_Z \varepsilon\right)' \varepsilon$ have order $O_p(N^{-1/2})$ as $N \to \infty$.

where $W(\cdot)$ denotes standard Brownian motion on $D[0,1]$.

Proof. By definition of the matrix T_1^{-1}, S_t is a unit root process with i.i.d. innovations ε_t. Also, using the particular form of the matrix X of deterministics, it is easy to obtain the following identities:

$$P_Z\varepsilon = \frac{1}{N-1}\left[(N-1)\,\varepsilon_1, S_N - \varepsilon_1, ..., S_N - \varepsilon_1\right]'$$

and

$$T_1^{-1}P_Z\varepsilon = \frac{1}{N-1}\left[(N-1)\,\varepsilon_1, S_{N-1} + (N-1)\,\varepsilon_1, ..., (N-1)\,S_{N-1} + (N-1)\,\varepsilon_1\right]'.$$

In what follows, we make use of standard unit root asymptotics, see e.g. Phillips (1987) and Phillips and Perron (1988).

For part (i), we have

$$\begin{aligned} N^{-1}S'\varepsilon &= N^{-1}\sum_{i=1}^{N} S_i\varepsilon_i = N^{-1}\left(\sum_{i=1}^{N} S_{i-1}\varepsilon_i + \sum_{i=1}^{N}\varepsilon_i^2\right) \\ &= N^{-1}\sum_{i=1}^{N} S_{i-1}\varepsilon_i + N^{-1}\sum_{i=1}^{N}\varepsilon_i^2 \\ &\Rightarrow \frac{1}{2}\sigma^2\left\{[W(1)]^2 - 1\right\} + \sigma^2 \\ &= \frac{1}{2}\sigma^2\left[W^2(1) + 1\right]. \end{aligned}$$

For part (ii),

$$\begin{aligned} \frac{1}{N}S'P_Z\varepsilon &= \frac{1}{N-1}\left[S_1\,(N-1)\,\varepsilon_1 + \sum_{i=2}^{N} S_i\,(S_N - \varepsilon_1)\right] \\ &= \frac{1}{N\,(N-1)}S_N\sum_{i=2}^{N} S_i + O_p\left(\frac{1}{N^2}\sum_{i=2}^{N} S_i\right) \\ &= \frac{1}{N^{1/2}}S_N\frac{1}{N^{3/2}}\sum_{i=2}^{N} S_i + O_p(N^{-1/2}) \\ &\Rightarrow \sigma^2 W(1)\int_0^1 W(r)dr. \end{aligned}$$

For part (iii),

$$
\begin{aligned}
\frac{1}{N^2} S' T_1^{-1} P_Z \varepsilon &= \frac{1}{N^2}\frac{1}{N-1}\sum_{i=1}^{N}\left\{S_i\left[(i-1)\,S_{N-1}+(N-1)\,\varepsilon_1\right]\right\} \\
&= \frac{1}{N^2}\frac{1}{N-1} S_{N-1}\sum_{i=1}^{N} S_i i + O_p\left(\frac{1}{N^2}\sum_{i=1}^{N} S_i\right) \\
&= \frac{1}{N^{1/2}} S_{N-1}\frac{1}{N^{5/2}}\sum_{i=1}^{N} S_i i + O_p\left(N^{-1/2}\right) \\
&\Rightarrow \sigma^2 W(1)\int_0^1 rW(r)dr.
\end{aligned}
$$

For part (iv),

$$
\begin{aligned}
\frac{1}{N^2}\left(T_1^{-1}P_Z\varepsilon\right)' T_1^{-1} P_Z \varepsilon &= \frac{1}{N^2}\left(\frac{1}{N-1}\right)^2\sum_{i=1}^{N}\left[(i-1)\,S_{N-1}+(N-1)\,\varepsilon_1\right]^2 \\
&= \frac{S_{N-1}^2}{(N-1)^2}\frac{1}{N^2}\sum_{i=1}^{N}(i-1)^2 + O_p\left(\frac{1}{N}S_{N-1}\right) \\
&= \left[1+o\left(1\right)\right]\frac{S_{N-1}^2}{3N} + O_p\left(N^{-1/2}\right) \\
&\Rightarrow \frac{1}{3}\sigma^2 W^2(1).
\end{aligned}
$$

For part (v),

$$
\begin{aligned}
\frac{1}{N}\left(T_1^{-1}P_Z\varepsilon\right)' P_Z \varepsilon &= \frac{1}{N}\left(\frac{1}{N-1}\right)^2\left\{(N-1)^2\,\varepsilon_1^2+\sum_{i=1}^{N-1}\left[\left(iS_{N-1}+(N-1)\,\varepsilon_1\right)\left(S_N-\varepsilon_1\right)\right]\right\} \\
&= \frac{1}{N\,(N-1)^2} S_{N-1} S_N\sum_{i=1}^{N-1} i + O_p\left(\frac{1}{N^3}S_{N-1}\sum_{i=1}^{N-1} i\right) \\
&= \left[1+o\left(1\right)\right]\frac{1}{2}\frac{S_{N-1}}{N^{1/2}}\frac{S_N}{N^{1/2}} + O_p\left(N^{-1/2}\right) \\
&\Rightarrow \frac{1}{2}\sigma^2 W^2(1)
\end{aligned}
$$

For part (vi),

$$\begin{aligned}
\frac{1}{N}\left(T_1^{-1}P_Z\varepsilon\right)'\varepsilon &= \frac{1}{N}\left\{\frac{1}{N-1}S_{N-1}\sum_{i=1}^{N}i\varepsilon_i - \frac{1}{N-1}S_{N-1}S_N + \varepsilon_1 S_N\right\} \\
&= \left[1+o\left(1\right)\right]\frac{S_{N-1}}{N^{1/2}}\frac{1}{N^{3/2}}\sum_{i=1}^{N}i\varepsilon_i + O_p\left(N^{-1/2}\right) \\
&\Rightarrow \sigma W(1)\left(\sigma W(1) - \sigma\int_0^1 W(r)dr\right).
\end{aligned}$$

For part (vii), recall that, under H_0, $Z = K_{\hat{\phi}}^{-1}T_1 X$ and $T_1 u = K_\phi \varepsilon$ which gives

$$\begin{aligned}
\nu &= M_Z K_{\hat{\phi}}^{-1}T_1\left(X\beta + u\right) = M_Z K_{\hat{\phi}}^{-1}T_1 u \\
&= M_Z K_{\hat{\phi}}^{-1}K_\phi\varepsilon = \left[I + o_p\left(1\right)\right]M_Z\varepsilon
\end{aligned}$$

using the fact that $\hat{\phi} - \phi = o_p\left(1\right).$ Therefore, since

$$\varepsilon' P_Z\varepsilon = \varepsilon_1^2 + \frac{1}{N-1}\left(S_N - \varepsilon_1\right)^2 = O_p\left(1\right),$$

the weak law of large numbers yields

$$\begin{aligned}
\frac{1}{N}\nu'\nu &= \left[I + o_p\left(1\right)\right]\frac{1}{N}\varepsilon' M_Z\varepsilon \\
&= \left[I + o_p\left(1\right)\right]\left\{\frac{1}{N}\varepsilon'\varepsilon + O_p\left(N^{-1}\right)\right\} \to_p \sigma^2.
\end{aligned}$$

For part (viii) P_Z corresponds to X including a constant term only which gives the following results,

$$\begin{aligned}
S' P_Z\varepsilon &= \left(T_1^{-1}P_Z\varepsilon\right)' P_Z\varepsilon = \varepsilon_1^2 = O_p(1), \\
\left(T_1^{-1}P_Z\varepsilon\right)' T_1^{-1}P_Z\varepsilon &= N\varepsilon_1^2 = O_p(N), \\
S' T_1^{-1}P_Z\varepsilon &= \varepsilon_1\sum_{i=1}^{N}S_i = O_p(N^{3/2}), \\
\left(T_1^{-1}P_Z\varepsilon\right)'\varepsilon &= \varepsilon_1\sum_{i=1}^{N}\varepsilon_i = \varepsilon_1 S_N = O_p(N^{1/2}),
\end{aligned}$$

A direct result from the above is that $N^{-1}S'P_Z\varepsilon$, $N^{-1}\left(T_1^{-1}P_Z\varepsilon\right)' P_Z\varepsilon$ and $N^{-2}\left(T_1^{-1}P_Z\varepsilon\right)' T_1^{-1}P_Z\varepsilon$ have order $O_p(N^{-1})$ and $N^{-2}S'T_1^{-1}P_Z\varepsilon$, $N^{-1}\left(T_1^{-1}P_Z\varepsilon\right)'\varepsilon$ have order $O_p(N^{-1/2})$ as $N \to \infty$.

Proof of Lemma 1. Using the commutation results given in Proposition A1 we get

$$\begin{aligned}\Sigma_{\rho,\phi} &= K_\phi^{-1} T_1 T_\rho^{-1} K_\phi K_\phi' \left(T_\rho^{-1}\right)' T_1' \left(K_\phi^{-1}\right)' = T_1 K_\phi^{-1} K_\phi T_\rho^{-1} \left(T_\rho^{-1}\right)' K_\phi' \left(K_\phi^{-1}\right)' T_1' \\ &= T_1 T_\rho^{-1} \left(T_\rho^{-1}\right)' T_1'.\end{aligned}$$

Proof of Theorem 2. The most powerful similar test of size α is given by (13) which can be rewritten as:

$$\frac{y' T_1' \left(K_\phi^{-1}\right)' C \left(C' \Sigma_\rho C\right)^{-1} C' K_\phi^{-1} T_1 y}{y' T_1' \left(K_\phi^{-1}\right)' M_Z K_\phi^{-1} T_1 y} < k_\alpha$$

Lemma 3 of Forchini and Marsh (2000) shows that the matrix

$$Q = C' B^{-1} C - \left(C' B C\right)^{-1}$$

is positive semi-definite. Applying this in our case gives the following result:

$$\begin{aligned}\frac{y' T_1' \left(K_\phi^{-1}\right)' C \left(C' \Sigma_\rho C\right)^{-1} C' K_\phi^{-1} T_1 y}{y' T_1' \left(K_\phi^{-1}\right)' M_Z K_\phi^{-1} T_1 y} &\leq \frac{y' T_1' \left(K_\phi^{-1}\right)' C C' \Sigma_\rho^{-1} C C' K_\phi^{-1} T_1 y}{y' T_1' \left(K_\phi^{-1}\right)' M_Z K_\phi^{-1} T_1 y} \\ \frac{y' T_1' \left(K_\phi^{-1}\right)' C \left(C' \Sigma_\rho C\right)^{-1} C' K_\phi^{-1} T_1 y}{y' T_1' \left(K_\phi^{-1}\right)' M_Z K_\phi^{-1} T_1 y} &\leq \frac{\nu' \Sigma_\rho^{-1} \nu}{\nu' \nu},\end{aligned}$$

where ν is defined above. So (13) is bounded above by the ratio of quadratic forms in ν. Inverting Σ_ρ and expressing T_ρ as $T_\rho = I_N - \rho L^{(1)}$:

$$\begin{gathered}\Sigma_\rho^{-1} = \left[T_1 T_\rho^{-1} \left(T_\rho^{-1}\right)' T_1'\right]^{-1} = \left(T_1^{-1}\right)' T_\rho' T_\rho T_1^{-1} = \\ \left(T_1^{-1}\right)' \left(I_N - \rho L^{(1)}\right)' \left(I_N - \rho L^{(1)}\right) T_1^{-1} = \\ \left(T_1^{-1}\right)' \left(I_N - \rho L^{(1)}\right) T_1^{-1} - \rho \left(T_1^{-1}\right)' L^{(1)\prime} \left(I_N - \rho L^{(1)}\right) T_1^{-1} = \\ \left(T_1^{-1}\right)' T_1^{-1} - \rho \left(T_1^{-1}\right)' L^{(1)} T_1^{-1} - \rho \left(T_1^{-1}\right)' L^{(1)\prime} T_1^{-1} + \rho^2 \left(T_1^{-1}\right)' L^{(1)\prime} L^{(1)} T_1^{-1}.\end{gathered} \tag{30}$$

From equation (30) and the definition of the matrix $\Psi(\nu)$ we obtain:

$$\frac{\nu' \Sigma_\rho^{-1} \nu}{\nu' \nu} = \begin{pmatrix} 1 & -\rho \end{pmatrix} \Psi(\nu) \begin{pmatrix} 1 \\ -\rho \end{pmatrix} \tag{31}$$

So a sufficient condition for (13) to hold is that the positive definite matrix $\Psi(\nu)$ is small with respect to some norm. We can find statistics such that $\Pr\{\|\Psi(\nu)\| < k_\alpha | H_0\} = a$.

Proof of Theorem 3. The first BEPO criterion is:

$$\frac{l(\rho^*)'\Psi(y)l(\rho^*)}{y'\Omega^{-1}(\rho_0)y} < k_a \tag{32}$$

where k_α is such that the size of the test is α and ρ^* is the value of ρ which minimizes (31). We differentiate (31) with respect to parameter ρ and set it equal to zero. From equations (18) and (31) we get:

$$\begin{aligned}
\begin{pmatrix} 1 & -\rho \end{pmatrix} \Psi(\nu) \begin{pmatrix} 1 \\ -\rho \end{pmatrix} &= \left(\rho^2\nu'\Psi_{22}\nu - 2\rho\nu'\Psi_{12}\nu + \nu'\Psi_{11}\nu\right). \\
\frac{1}{\nu'\nu}\frac{\partial\left(\rho^{*2}\nu'\Psi_{22}\nu - 2\rho^*\nu'\Psi_{12}\nu + \nu'\Psi_{11}\nu\right)}{\partial\rho^*} &= 0 \Rightarrow \\
\frac{2}{\nu'\nu}\left(\rho^*\psi_{22} - \psi_{12}\right) &= 0 \Rightarrow \rho^* = \frac{\psi_{12}}{\psi_{22}}. \qquad (33)
\end{aligned}$$

Combining condition (32) with (33) and values given by (16) and (17) we get the BEPO statistic. Also we need to note that $\psi_{22} \geq 0$ since Ψ_{22} is a positive semi-definite matrix, so $\frac{\partial^2\left(\rho^2\psi_{22} - 2\rho\psi_{12} + \psi_{11}\right)}{\partial\rho^2} \geq 0$.

The theorem is proved by substituting (33) and in (3).

Proof of Theorem 4. We make repeated use of the limit theory established in Proposition A2. For notational simplicity, define

$$\psi_{11} = \nu'\Psi_{11}\nu,\ \psi_{22} = \nu'\Psi_{22}\nu \text{ and } \psi_{12} = \nu'\Psi_{12}\nu$$

and note that

$$\begin{aligned}
\psi_{22} &= \psi_{11} - 2\left(T_1^{-1}\nu\right)'\nu + \nu'\nu \\
&= \psi_{11} - 2\left[S'\varepsilon - S'P_Z\varepsilon + \left(T_1^{-1}P_Z\varepsilon\right)'P_Z\varepsilon\right] - 2\left(T_1^{-1}P_Z\varepsilon\right)'\varepsilon + \nu'\nu \qquad (34)
\end{aligned}$$

and

$$\psi_{12} = \psi_{11} - S'\varepsilon + S'P_Z\varepsilon - \left(T_1^{-1}P_Z\varepsilon\right)'P_Z\varepsilon. \tag{35}$$

For part (i), it is clear Proposition A2 and (34) and (35) we obtain that $\psi_{22} = \psi_{11} + O_p(N)$ and $\psi_{12} = \psi_{11} + O_p(N)$. Now by Proposition A2,

$$\begin{aligned}
\frac{1}{N^2}\psi_{11} &= \frac{1}{N^2}S'S - \frac{2}{N^2}S'T_1^{-1}P_Z\varepsilon + \frac{1}{N^2}\left(T_1^{-1}P_Z\varepsilon\right)'T_1^{-1}P_Z\varepsilon \\
&\Rightarrow \sigma^2\left\{\int_0^1 W^2(r)dr - 2W(1)\int_0^1 rW(r)dr + \frac{1}{3}W^2(1)\right\}. \qquad (36)
\end{aligned}$$

The BNM test statistic is given by

$$\begin{aligned}
\frac{1}{N}\left\|\Psi(\nu)\right\| &= \frac{1}{N^{-1}\nu'\nu}\left\|\frac{1}{N^2}\begin{bmatrix}\psi_{11} & \psi_{12}\\ \psi_{12} & \psi_{22}\end{bmatrix}\right\| \\
&= \frac{1}{N^{-1}\nu'\nu}\left\|\frac{1}{N^2}\begin{bmatrix}\psi_{11} & \psi_{12}\\ \psi_{12} & \psi_{22}\end{bmatrix}\right\| \\
&= \frac{1}{N^{-1}\nu'\nu}\left\|\frac{\psi_{11}}{N^2}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\right\| + O_p\left(N^{-1}\right) \\
&\Rightarrow \left\|\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\right\|\left\{\int_0^1 W^2(r)dr - 2W(1)\int_0^1 rW(r)dr + \frac{1}{3}W^2(1)\right\}
\end{aligned}$$

and the result follows from Proposition A2(vi) and (36).

Part (iii) corresponds to the case of a constant only included in the model. Proposition A2(viii) applies here and we get

$$\frac{1}{N^2}\psi_{11} = \frac{1}{N^2}S'S + O_p(N^{-1/2}) \Rightarrow \sigma^2\int_0^1 W^2(r)dr. \tag{37}$$

The above result in conjunction with Proposition A2(vii) gives us

$$\frac{1}{N}\left\|\Psi(\nu)\right\| = \frac{1}{N^{-1}\nu'\nu}\left\|\frac{1}{N^2}\begin{bmatrix}\psi_{11} & \psi_{12}\\ \psi_{12} & \psi_{22}\end{bmatrix}\right\| \Rightarrow \left\|\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\right\|\int_0^1 W^2(r)dr.$$

For part (ii):

$$\begin{aligned}
\frac{1}{N}\left(\psi_{12}-\psi_{22}\right) &= \frac{1}{N}\left(S'\varepsilon - S'P_Z\varepsilon - \left(T_1^{-1}P_Z\varepsilon\right)'\varepsilon + \left(T_1^{-1}P_Z\varepsilon\right)'P_Z\varepsilon - \nu'\nu\right) \\
&\xrightarrow{L} \frac{1}{2}\sigma^2\left[W^2(1)+1\right] - \sigma^2 W(1)\int_0^1 W(r)dr \\
&\quad -\sigma^2 W(1)\left(W(1) - \int_0^1 W(r)dr\right) + \frac{1}{2}\sigma^2 W^2(1) - \sigma^2 \\
&= \sigma^2\left\{\begin{array}{c}\frac{1}{2}\left[W^2(1)+1\right] - W(1)\int_0^1 W(r)dr \\ -W(1)\left(W(1) - \int_0^1 W(r)dr\right) + \frac{1}{2}W^2(1) - 1\end{array}\right\} \\
&= \sigma^2\left\{\begin{array}{c}\frac{1}{2}W^2(1) + \frac{1}{2} - W(1)\int_0^1 W(r)dr \\ -W^2(1) + W(1)\int_0^1 W(r)dr + \frac{1}{2}W^2(1) - 1\end{array}\right\} = -\frac{1}{2}\sigma^2
\end{aligned}$$

As before, when a constant and trend are included in the model $N^{-2}\psi_{22} = N^{-2}\psi_{11} + O_p(N^{-1})$. Combining the above results and the one in (36), we get the asymptotic distribution of BEPO statistic which is given by

$$\begin{aligned} BEPO &= N\left|\frac{\psi_{12}-\psi_{22}}{\psi_{22}}\right| = \left|\frac{N^{-1}(\psi_{12}-\psi_{22})}{N^{-2}\psi_{22}}\right| \\ &\xrightarrow{L} \left|\frac{-\frac{1}{2}\sigma^2}{\sigma^2\left[\int_0^1 W^2(r)dr - 2W(1)\int_0^1 rW(r)dr + \frac{1}{3}W^2(1)\right]}\right| \\ &= \left|\frac{-\frac{1}{2}}{\int_0^1 W^2(r)dr - 2W(1)\int_0^1 rW(r)dr + \frac{1}{3}W^2(1)}\right| \\ &= \frac{1}{2}\frac{1}{\left|\int_0^1 W^2(r)dr - 2W(1)\int_0^1 rW(r)dr + \frac{1}{3}W^2(1)\right|}. \end{aligned}$$

For part (iv) of the theorem X satisfies (23). We use results from Proposition A2(viii) and we get

$$\begin{aligned} \frac{1}{N}(\psi_{12}-\psi_{22}) &= \frac{1}{N}\left(S'\varepsilon - S'P_Z\varepsilon - \left(T_1^{-1}P_Z\varepsilon\right)'\varepsilon + \left(T_1^{-1}P_Z\varepsilon\right)'P_Z\varepsilon - \nu'\nu\right) \\ &= \frac{1}{N}(S'\varepsilon - \nu'\nu) + O_p\left(N^{-1/2}\right) \\ \xrightarrow{L} \frac{1}{2}\sigma^2\left[W^2(1)+1\right] - \sigma^2 &= \frac{1}{2}\sigma^2\left[W^2(1)-1\right]. \end{aligned}$$

Using the above result and (37) we get

$$\begin{aligned} BEPO &= N\left|\frac{\psi_{12}-\psi_{22}}{\psi_{22}}\right| = \left|\frac{N^{-1}(\psi_{12}-\psi_{22})}{N^{-2}\psi_{22}}\right| \\ \xrightarrow{L} \left|\frac{\frac{1}{2}\sigma^2\left[W^2(1)-1\right]}{\sigma^2\int_0^1 W^2(r)dr}\right| &= \frac{1}{2}\left|\frac{W^2(1)-1}{\int_0^1 W^2(r)dr}\right|. \end{aligned}$$

9 Tables and Figures

Table 1a. Empirical size of the tests for model with an intercept only

N	ϕ	BNM_0	$BEPO_0$	BNM_b	$BEPO_b$	BNM_a	$BEPO_a$	BNM_h	$BEPO_h$	MZ_a	MZ_t	MSB	P_t	MP_t	ADF	MZ_{aLS}	ADF_{LS}	MZ_{a2}
50	−0.8	0.669	0.673	0.496	0.501	0.367	0.367	0.433	0.437	0.992	0.988	0.992	0.931	0.986	0.997	0.975	0.992	0.976
	−0.7	0.559	0.563	0.370	0.367	0.253	0.251	0.298	0.305	0.932	0.916	0.930	0.807	0.909	0.957	0.831	0.918	0.903
	−0.6	0.464	0.466	0.261	0.260	0.179	0.176	0.196	0.196	0.791	0.762	0.783	0.637	0.751	0.842	0.599	0.759	0.751
	−0.5	0.364	0.366	0.215	0.214	0.149	0.148	0.163	0.164	0.605	0.573	0.593	0.454	0.564	0.680	0.389	0.568	0.569
	−0.4	0.255	0.259	0.175	0.170	0.112	0.115	0.127	0.130	0.435	0.410	0.425	0.319	0.399	0.520	0.220	0.389	0.407
	−0.3	0.155	0.162	0.152	0.154	0.081	0.084	0.096	0.102	0.292	0.271	0.281	0.213	0.267	0.364	0.111	0.247	0.271
	−0.2	0.135	0.135	0.104	0.101	0.077	0.076	0.077	0.076	0.181	0.164	0.172	0.128	0.162	0.237	0.052	0.157	0.167
	−0.1	0.068	0.070	0.076	0.073	0.064	0.064	0.060	0.063	0.104	0.095	0.097	0.075	0.095	0.149	0.022	0.098	0.097
	0	0.050	0.054	0.049	0.045	0.047	0.046	0.049	0.050	0.062	0.056	0.058	0.044	0.056	0.096	0.008	0.057	0.057
	0.1	0.023	0.024	0.049	0.045	0.047	0.045	0.046	0.047	0.041	0.038	0.038	0.031	0.037	0.068	0.003	0.046	0.038
	0.2	0.025	0.027	0.052	0.048	0.048	0.047	0.047	0.048	0.025	0.023	0.025	0.019	0.023	0.045	0.001	0.037	0.023
	0.3	0.018	0.019	0.047	0.047	0.056	0.056	0.045	0.046	0.016	0.014	0.015	0.011	0.013	0.028	0.001	0.031	0.015
	0.4	0.010	0.010	0.057	0.057	0.050	0.050	0.050	0.048	0.011	0.009	0.010	0.007	0.009	0.021	0.000	0.029	0.010
	0.5	0.011	0.010	0.064	0.061	0.052	0.052	0.044	0.045	0.009	0.009	0.007	0.007	0.008	0.018	0.000	0.030	0.008
	0.6	0.007	0.007	0.062	0.061	0.039	0.038	0.039	0.041	0.008	0.007	0.006	0.006	0.007	0.015	0.000	0.029	0.007
	0.7	0.008	0.009	0.053	0.053	0.039	0.039	0.035	0.039	0.005	0.005	0.005	0.005	0.005	0.014	0.000	0.027	0.005
	0.8	0.009	0.008	0.052	0.050	0.039	0.040	0.048	0.050	0.005	0.005	0.004	0.004	0.005	0.011	0.000	0.033	0.005
100	−0.8	0.831	0.832	0.244	0.247	0.220	0.220	0.224	0.227	0.042	0.039	0.035	0.037	0.039	0.120	0.068	0.113	0.096
	−0.7	0.680	0.681	0.146	0.145	0.131	0.130	0.140	0.141	0.038	0.036	0.032	0.034	0.037	0.087	0.035	0.071	0.066
	−0.6	0.566	0.569	0.103	0.103	0.105	0.105	0.100	0.100	0.052	0.050	0.046	0.046	0.050	0.084	0.029	0.063	0.070
	−0.5	0.435	0.436	0.090	0.090	0.086	0.085	0.086	0.087	0.060	0.057	0.054	0.050	0.057	0.079	0.031	0.059	0.074
	−0.4	0.327	0.328	0.101	0.099	0.077	0.077	0.091	0.093	0.059	0.054	0.055	0.049	0.055	0.068	0.029	0.053	0.069
	−0.3	0.210	0.213	0.099	0.099	0.076	0.077	0.090	0.090	0.058	0.054	0.053	0.047	0.053	0.063	0.025	0.047	0.063
	−0.2	0.135	0.133	0.103	0.102	0.072	0.071	0.086	0.088	0.059	0.054	0.055	0.046	0.054	0.063	0.025	0.052	0.063
	−0.1	0.071	0.071	0.074	0.073	0.059	0.060	0.075	0.078	0.059	0.056	0.054	0.048	0.055	0.066	0.022	0.048	0.061
	0	0.053	0.054	0.051	0.049	0.049	0.050	0.050	0.051	0.047	0.044	0.044	0.038	0.043	0.053	0.015	0.042	0.048
	0.1	0.036	0.035	0.038	0.037	0.046	0.046	0.046	0.047	0.039	0.036	0.038	0.031	0.036	0.042	0.010	0.037	0.039
	0.2	0.010	0.011	0.041	0.041	0.052	0.051	0.053	0.053	0.042	0.037	0.038	0.032	0.038	0.039	0.010	0.034	0.041
	0.3	0.021	0.021	0.049	0.050	0.056	0.055	0.059	0.058	0.058	0.054	0.053	0.047	0.053	0.053	0.020	0.034	0.058
	0.4	0.008	0.008	0.053	0.052	0.053	0.053	0.053	0.054	0.065	0.060	0.057	0.051	0.060	0.056	0.025	0.039	0.067
	0.5	0.004	0.004	0.055	0.055	0.053	0.052	0.057	0.059	0.054	0.050	0.048	0.044	0.051	0.045	0.025	0.038	0.059
	0.6	0.005	0.005	0.050	0.049	0.054	0.054	0.055	0.054	0.059	0.054	0.056	0.047	0.052	0.044	0.031	0.036	0.066
	0.7	0.006	0.006	0.052	0.053	0.049	0.048	0.055	0.057	0.060	0.055	0.054	0.049	0.054	0.041	0.031	0.031	0.064
	0.8	0.006	0.005	0.052	0.052	0.048	0.048	0.052	0.053	0.073	0.068	0.068	0.058	0.066	0.044	0.041	0.026	0.081

Table 1b. Empirical size of the tests for model with an intercept only

N	ϕ	BNM_0	$BEPO_0$	BNM_b	$BEPO_b$	BNM_a	$BEPO_a$	BNM_h	$BEPO_h$	MZ_a	MZ_t	MSB	P_t	MP_t	ADF	MZ_{aLS}	ADF_{LS}	MZ_{a2}
200	−0.8	0.908	0.908	0.097	0.098	0.101	0.101	0.104	0.102	0.023	0.021	0.019	0.021	0.022	0.085	0.021	0.070	0.042
	−0.7	0.768	0.767	0.076	0.078	0.073	0.076	0.083	0.082	0.039	0.038	0.033	0.036	0.038	0.075	0.023	0.059	0.050
	−0.6	0.598	0.599	0.065	0.066	0.067	0.068	0.073	0.072	0.049	0.047	0.042	0.044	0.047	0.069	0.027	0.053	0.057
	−0.5	0.481	0.480	0.057	0.057	0.061	0.062	0.065	0.062	0.056	0.055	0.049	0.051	0.055	0.064	0.029	0.051	0.061
	−0.4	0.346	0.346	0.055	0.057	0.055	0.057	0.066	0.065	0.054	0.050	0.048	0.047	0.051	0.057	0.030	0.046	0.059
	−0.3	0.202	0.202	0.061	0.061	0.052	0.052	0.058	0.055	0.060	0.056	0.055	0.051	0.055	0.059	0.034	0.051	0.063
	−0.2	0.151	0.151	0.071	0.071	0.054	0.056	0.067	0.066	0.052	0.049	0.046	0.045	0.048	0.052	0.028	0.049	0.055
	−0.1	0.085	0.086	0.070	0.074	0.057	0.057	0.066	0.065	0.053	0.050	0.050	0.045	0.049	0.053	0.028	0.047	0.055
	0	0.048	0.048	0.045	0.044	0.047	0.047	0.049	0.050	0.051	0.046	0.046	0.042	0.046	0.050	0.022	0.046	0.052
	0.1	0.033	0.033	0.038	0.040	0.048	0.047	0.046	0.044	0.045	0.041	0.040	0.037	0.041	0.043	0.015	0.037	0.044
	0.2	0.014	0.015	0.043	0.045	0.052	0.052	0.054	0.052	0.052	0.048	0.048	0.044	0.048	0.048	0.022	0.040	0.053
	0.3	0.005	0.005	0.048	0.048	0.047	0.046	0.052	0.052	0.058	0.054	0.051	0.047	0.053	0.052	0.031	0.047	0.059
	0.4	0.009	0.009	0.045	0.046	0.051	0.050	0.052	0.050	0.050	0.046	0.048	0.042	0.046	0.044	0.028	0.042	0.051
	0.5	0.006	0.006	0.045	0.046	0.046	0.047	0.052	0.053	0.050	0.046	0.048	0.042	0.046	0.041	0.027	0.038	0.053
	0.6	0.012	0.012	0.042	0.044	0.048	0.049	0.052	0.052	0.054	0.050	0.052	0.044	0.049	0.042	0.031	0.036	0.059
	0.7	0.009	0.009	0.042	0.043	0.046	0.046	0.047	0.047	0.059	0.053	0.054	0.048	0.052	0.041	0.038	0.037	0.064
	0.8	0.009	0.008	0.048	0.049	0.049	0.050	0.052	0.051	0.065	0.063	0.059	0.058	0.063	0.045	0.043	0.031	0.075
400	−0.8	0.936	0.947	0.074	0.077	0.073	0.070	0.072	0.073	0.028	0.027	0.022	0.026	0.027	0.085	0.012	0.066	0.035
	−0.7	0.781	0.793	0.063	0.064	0.062	0.061	0.064	0.066	0.046	0.044	0.038	0.041	0.044	0.066	0.021	0.061	0.050
	−0.6	0.625	0.633	0.060	0.064	0.055	0.055	0.059	0.059	0.052	0.049	0.044	0.047	0.048	0.061	0.033	0.059	0.056
	−0.5	0.456	0.480	0.054	0.055	0.058	0.056	0.062	0.061	0.052	0.049	0.047	0.047	0.049	0.054	0.033	0.052	0.054
	−0.4	0.275	0.285	0.060	0.061	0.054	0.053	0.057	0.055	0.054	0.051	0.050	0.048	0.050	0.051	0.037	0.052	0.055
	−0.3	0.179	0.182	0.054	0.055	0.050	0.049	0.054	0.054	0.051	0.049	0.048	0.047	0.049	0.050	0.031	0.049	0.052
	−0.2	0.096	0.115	0.056	0.057	0.051	0.049	0.057	0.055	0.056	0.052	0.049	0.048	0.051	0.053	0.034	0.050	0.057
	−0.1	0.066	0.086	0.070	0.069	0.053	0.053	0.063	0.063	0.051	0.047	0.050	0.045	0.048	0.049	0.032	0.049	0.052
	0	0.031	0.040	0.052	0.053	0.049	0.049	0.053	0.054	0.047	0.043	0.043	0.040	0.043	0.044	0.025	0.042	0.047
	0.1	0.016	0.022	0.041	0.042	0.049	0.049	0.046	0.048	0.053	0.050	0.050	0.047	0.049	0.051	0.029	0.046	0.053
	0.2	0.008	0.010	0.051	0.053	0.051	0.049	0.049	0.050	0.052	0.047	0.047	0.044	0.046	0.047	0.032	0.045	0.052
	0.3	0.012	0.012	0.053	0.053	0.055	0.053	0.055	0.056	0.052	0.048	0.049	0.045	0.047	0.047	0.033	0.045	0.054
	0.4	0.004	0.006	0.053	0.054	0.050	0.048	0.051	0.052	0.052	0.048	0.047	0.045	0.046	0.046	0.029	0.043	0.053
	0.5	0.003	0.008	0.051	0.053	0.047	0.047	0.053	0.053	0.051	0.048	0.045	0.045	0.047	0.044	0.032	0.042	0.053
	0.6	0.002	0.001	0.055	0.056	0.051	0.049	0.048	0.048	0.055	0.050	0.050	0.048	0.050	0.045	0.034	0.042	0.057
	0.7	0.002	0.002	0.049	0.052	0.049	0.047	0.045	0.046	0.053	0.051	0.050	0.047	0.050	0.043	0.040	0.042	0.058
	0.8	0.004	0.003	0.052	0.050	0.049	0.047	0.053	0.052	0.059	0.055	0.054	0.053	0.054	0.045	0.045	0.039	0.064

Table 2a. Empirical size of the tests for model with an intercept and a trend

N	ϕ	BNM_0	$BEPO_0$	BNM_b	$BEPO_b$	BNM_a	$BEPO_a$	BNM_h	$BEPO_h$	MZ_a	MZ_t	MSB	P_t	MP_t	ADF	MZ_{aLS}	ADF_{LS}	MZ_{a2}
50	−0.8	0.778	0.781	0.727	0.726	0.532	0.533	0.643	0.644	0.993	0.991	0.994	0.957	0.988	1.000	0.910	1.000	0.957
	−0.7	0.717	0.717	0.620	0.618	0.437	0.438	0.534	0.535	0.931	0.924	0.933	0.864	0.917	0.995	0.716	0.989	0.876
	−0.6	0.613	0.613	0.491	0.489	0.329	0.329	0.415	0.416	0.761	0.749	0.768	0.693	0.745	0.959	0.453	0.922	0.699
	−0.5	0.456	0.457	0.375	0.372	0.237	0.237	0.307	0.307	0.523	0.514	0.531	0.473	0.511	0.840	0.234	0.771	0.469
	−0.4	0.337	0.341	0.267	0.264	0.175	0.176	0.215	0.216	0.306	0.300	0.315	0.281	0.302	0.671	0.096	0.579	0.270
	−0.3	0.224	0.225	0.180	0.178	0.119	0.119	0.143	0.143	0.160	0.155	0.167	0.148	0.157	0.468	0.032	0.382	0.134
	−0.2	0.139	0.139	0.112	0.110	0.072	0.072	0.096	0.097	0.071	0.069	0.075	0.068	0.071	0.298	0.012	0.233	0.058
	−0.1	0.069	0.071	0.069	0.069	0.067	0.067	0.066	0.066	0.028	0.027	0.031	0.028	0.028	0.176	0.002	0.132	0.023
	0	0.043	0.044	0.048	0.048	0.053	0.053	0.057	0.056	0.010	0.010	0.010	0.011	0.011	0.090	0.001	0.068	0.009
	0.1	0.025	0.026	0.042	0.041	0.053	0.052	0.050	0.051	0.005	0.005	0.005	0.006	0.005	0.056	0.000	0.045	0.004
	0.2	0.015	0.015	0.040	0.038	0.060	0.059	0.055	0.055	0.001	0.001	0.002	0.002	0.002	0.030	0.000	0.028	0.001
	0.3	0.003	0.003	0.054	0.053	0.058	0.057	0.056	0.056	0.001	0.001	0.001	0.001	0.001	0.018	0.000	0.020	0.001
	0.4	0.006	0.006	0.051	0.050	0.051	0.051	0.060	0.060	0.000	0.000	0.000	0.000	0.000	0.011	0.000	0.016	0.000
	0.5	0.007	0.007	0.057	0.055	0.049	0.049	0.056	0.056	0.000	0.000	0.000	0.000	0.000	0.007	0.000	0.012	0.000
	0.6	0.002	0.002	0.049	0.049	0.040	0.038	0.054	0.054	0.000	0.000	0.000	0.000	0.000	0.005	0.000	0.013	0.000
	0.7	0.002	0.002	0.050	0.050	0.047	0.047	0.054	0.054	0.000	0.000	0.000	0.000	0.000	0.004	0.000	0.008	0.000
	0.8	0.001	0.001	0.049	0.048	0.039	0.038	0.049	0.049	0.000	0.000	0.000	0.000	0.000	0.004	0.000	0.011	0.000
100	−0.8	0.931	0.931	0.595	0.596	0.465	0.463	0.509	0.509	0.067	0.068	0.067	0.070	0.069	0.127	0.143	0.196	0.163
	−0.7	0.841	0.838	0.343	0.344	0.262	0.260	0.281	0.281	0.040	0.040	0.040	0.042	0.042	0.089	0.073	0.116	0.091
	−0.6	0.743	0.742	0.211	0.212	0.180	0.178	0.180	0.180	0.036	0.037	0.036	0.037	0.037	0.076	0.045	0.078	0.064
	−0.5	0.581	0.579	0.185	0.186	0.140	0.138	0.146	0.144	0.035	0.035	0.035	0.035	0.036	0.070	0.036	0.065	0.052
	−0.4	0.380	0.378	0.177	0.177	0.111	0.110	0.135	0.134	0.038	0.038	0.039	0.039	0.040	0.066	0.032	0.061	0.051
	−0.3	0.272	0.269	0.161	0.162	0.103	0.102	0.115	0.115	0.034	0.034	0.035	0.034	0.035	0.059	0.027	0.054	0.043
	−0.2	0.172	0.167	0.128	0.129	0.085	0.084	0.098	0.098	0.033	0.033	0.034	0.032	0.033	0.057	0.023	0.052	0.039
	−0.1	0.089	0.088	0.080	0.080	0.062	0.061	0.068	0.067	0.028	0.028	0.028	0.026	0.029	0.054	0.016	0.054	0.031
	0	0.050	0.050	0.046	0.047	0.053	0.052	0.045	0.044	0.020	0.021	0.020	0.021	0.022	0.043	0.010	0.042	0.021
	0.1	0.028	0.027	0.041	0.041	0.048	0.047	0.044	0.043	0.011	0.011	0.012	0.011	0.012	0.025	0.005	0.029	0.010
	0.2	0.010	0.010	0.043	0.044	0.055	0.054	0.050	0.049	0.012	0.012	0.013	0.011	0.012	0.019	0.004	0.018	0.012
	0.3	0.005	0.005	0.061	0.062	0.056	0.055	0.050	0.050	0.020	0.020	0.022	0.020	0.021	0.023	0.011	0.022	0.020
	0.4	0.006	0.006	0.060	0.061	0.055	0.054	0.047	0.047	0.031	0.029	0.033	0.028	0.030	0.026	0.020	0.023	0.034
	0.5	0.004	0.004	0.050	0.051	0.049	0.048	0.046	0.045	0.041	0.040	0.042	0.039	0.040	0.032	0.028	0.026	0.045
	0.6	0.006	0.006	0.056	0.056	0.051	0.051	0.048	0.048	0.040	0.040	0.042	0.038	0.040	0.027	0.034	0.023	0.050
	0.7	0.002	0.002	0.054	0.053	0.050	0.049	0.044	0.044	0.043	0.041	0.044	0.041	0.042	0.021	0.035	0.019	0.049
	0.8	0.001	0.000	0.047	0.047	0.045	0.044	0.047	0.047	0.056	0.054	0.059	0.052	0.054	0.018	0.048	0.015	0.067

Table 2b. Empirical size of the tests for model with an intercept and a trend

N	ϕ	BNM_0	$BEPO_0$	BNM_b	$BEPO_b$	BNM_a	$BEPO_a$	BNM_h	$BEPO_h$	MZ_a	MZ_t	MSB	P_t	MP_t	ADF	MZ_{aLS}	ADF_{LS}	MZ_{a2}
200	−0.8	0.983	0.983	0.187	0.187	0.190	0.191	0.189	0.188	0.015	0.015	0.015	0.016	0.016	0.074	0.042	0.095	0.049
	−0.7	0.921	0.921	0.112	0.112	0.113	0.114	0.122	0.120	0.021	0.021	0.021	0.022	0.022	0.061	0.032	0.071	0.042
	−0.6	0.796	0.796	0.095	0.095	0.088	0.088	0.098	0.096	0.026	0.026	0.026	0.026	0.027	0.052	0.031	0.058	0.039
	−0.5	0.625	0.624	0.076	0.076	0.076	0.077	0.084	0.082	0.032	0.033	0.032	0.034	0.034	0.051	0.035	0.054	0.043
	−0.4	0.458	0.457	0.069	0.069	0.067	0.068	0.073	0.072	0.031	0.030	0.032	0.031	0.032	0.043	0.030	0.047	0.039
	−0.3	0.284	0.283	0.084	0.084	0.068	0.069	0.077	0.075	0.040	0.040	0.040	0.039	0.041	0.048	0.034	0.050	0.045
	−0.2	0.161	0.161	0.098	0.098	0.069	0.069	0.083	0.082	0.035	0.035	0.036	0.035	0.037	0.046	0.031	0.046	0.040
	−0.1	0.086	0.085	0.084	0.084	0.059	0.059	0.076	0.075	0.034	0.033	0.034	0.032	0.034	0.042	0.026	0.043	0.036
	0	0.055	0.055	0.046	0.046	0.047	0.047	0.053	0.052	0.027	0.027	0.028	0.027	0.028	0.039	0.020	0.042	0.028
	0.1	0.026	0.026	0.036	0.036	0.049	0.049	0.046	0.046	0.017	0.017	0.018	0.016	0.017	0.022	0.011	0.026	0.016
	0.2	0.013	0.013	0.051	0.051	0.047	0.048	0.053	0.051	0.025	0.025	0.026	0.024	0.025	0.028	0.016	0.027	0.025
	0.3	0.011	0.011	0.050	0.050	0.054	0.054	0.053	0.052	0.037	0.035	0.038	0.034	0.035	0.035	0.030	0.036	0.038
	0.4	0.003	0.003	0.052	0.052	0.050	0.050	0.054	0.053	0.037	0.036	0.038	0.035	0.036	0.033	0.033	0.033	0.042
	0.5	0.004	0.004	0.049	0.049	0.046	0.047	0.055	0.054	0.032	0.032	0.034	0.031	0.032	0.025	0.027	0.027	0.036
	0.6	0.002	0.002	0.046	0.046	0.048	0.049	0.053	0.052	0.039	0.038	0.040	0.037	0.038	0.027	0.033	0.024	0.043
	0.7	0.000	0.000	0.049	0.049	0.047	0.048	0.051	0.050	0.046	0.046	0.048	0.044	0.046	0.023	0.040	0.024	0.050
	0.8	0.004	0.004	0.048	0.048	0.049	0.050	0.052	0.051	0.055	0.054	0.056	0.052	0.053	0.018	0.054	0.019	0.065
400	−0.8	0.999	0.999	0.101	0.102	0.104	0.102	0.103	0.103	0.010	0.011	0.010	0.011	0.011	0.062	0.014	0.071	0.019
	−0.7	0.956	0.956	0.078	0.078	0.076	0.075	0.086	0.085	0.017	0.017	0.017	0.019	0.019	0.053	0.020	0.054	0.024
	−0.6	0.839	0.839	0.062	0.062	0.071	0.070	0.073	0.072	0.031	0.032	0.031	0.032	0.032	0.051	0.031	0.055	0.037
	−0.5	0.650	0.650	0.061	0.061	0.063	0.061	0.064	0.063	0.034	0.035	0.036	0.035	0.035	0.045	0.032	0.048	0.040
	−0.4	0.487	0.487	0.056	0.056	0.061	0.060	0.056	0.056	0.039	0.040	0.040	0.040	0.041	0.047	0.035	0.048	0.044
	−0.3	0.307	0.309	0.052	0.052	0.059	0.058	0.058	0.058	0.041	0.042	0.041	0.041	0.043	0.045	0.034	0.048	0.043
	−0.2	0.205	0.205	0.067	0.067	0.055	0.054	0.056	0.056	0.038	0.037	0.037	0.037	0.038	0.040	0.031	0.041	0.039
	−0.1	0.115	0.115	0.081	0.081	0.060	0.059	0.069	0.068	0.035	0.036	0.036	0.036	0.036	0.039	0.030	0.040	0.037
	0	0.055	0.056	0.048	0.049	0.046	0.046	0.051	0.050	0.036	0.035	0.037	0.034	0.036	0.040	0.028	0.043	0.036
	0.1	0.039	0.039	0.038	0.038	0.050	0.050	0.051	0.050	0.028	0.028	0.028	0.028	0.029	0.029	0.019	0.033	0.027
	0.2	0.015	0.015	0.055	0.055	0.051	0.051	0.051	0.050	0.039	0.039	0.040	0.038	0.039	0.039	0.036	0.043	0.041
	0.3	0.011	0.011	0.047	0.047	0.052	0.051	0.054	0.054	0.037	0.037	0.039	0.037	0.038	0.035	0.032	0.037	0.039
	0.4	0.007	0.007	0.045	0.045	0.047	0.046	0.055	0.055	0.035	0.034	0.035	0.033	0.034	0.031	0.034	0.035	0.036
	0.5	0.005	0.005	0.046	0.046	0.052	0.050	0.054	0.053	0.038	0.037	0.040	0.038	0.039	0.030	0.034	0.033	0.041
	0.6	0.002	0.002	0.045	0.045	0.051	0.050	0.051	0.050	0.041	0.040	0.041	0.041	0.042	0.032	0.040	0.035	0.044
	0.7	0.003	0.003	0.050	0.050	0.046	0.045	0.054	0.053	0.045	0.045	0.046	0.044	0.046	0.030	0.046	0.031	0.051
	0.8	0.003	0.003	0.044	0.044	0.046	0.046	0.052	0.052	0.053	0.052	0.055	0.051	0.053	0.030	0.059	0.031	0.061

Table 3. Power of the tests for model with an intercept only, $\phi = 0$

N	ρ	BNM_0	$BEPO_0$	BNM_b	$BEPO_b$	BNM_a	$BEPO_a$	BNM_h	$BEPO_h$	MZ_a	MZ_t	MSB	P_t	MP_t	ADF	MZ_{aLS}	ADF_{LS}	MZ_{a2}
50	0.80	0.550	0.566	0.530	0.542	0.336	0.338	0.408	0.424	0.772	0.753	0.713	0.737	0.748	0.862	0.216	0.383	0.769
	0.82	0.486	0.495	0.484	0.488	0.312	0.313	0.374	0.388	0.702	0.684	0.640	0.657	0.680	0.812	0.178	0.324	0.696
	0.84	0.430	0.440	0.419	0.425	0.268	0.268	0.312	0.326	0.614	0.591	0.548	0.564	0.589	0.741	0.139	0.266	0.605
	0.86	0.350	0.358	0.346	0.355	0.239	0.237	0.264	0.275	0.536	0.515	0.471	0.481	0.513	0.666	0.110	0.221	0.526
	0.88	0.282	0.292	0.298	0.299	0.207	0.206	0.234	0.243	0.448	0.434	0.383	0.395	0.430	0.580	0.083	0.181	0.435
	0.90	0.211	0.222	0.241	0.244	0.179	0.179	0.188	0.196	0.366	0.346	0.319	0.309	0.345	0.488	0.061	0.141	0.355
	0.92	0.184	0.187	0.195	0.195	0.155	0.155	0.157	0.163	0.283	0.269	0.249	0.229	0.269	0.392	0.046	0.121	0.271
	0.94	0.125	0.128	0.149	0.148	0.120	0.120	0.121	0.126	0.213	0.198	0.182	0.164	0.198	0.304	0.033	0.106	0.200
	0.96	0.100	0.103	0.112	0.112	0.101	0.100	0.093	0.093	0.142	0.131	0.120	0.105	0.131	0.215	0.020	0.084	0.131
	0.98	0.064	0.068	0.079	0.078	0.074	0.073	0.066	0.066	0.094	0.086	0.086	0.068	0.085	0.146	0.013	0.067	0.087
100	0.80	0.926	0.930	0.913	0.922	0.874	0.882	0.901	0.911	0.841	0.836	0.823	0.831	0.834	0.855	0.646	0.597	0.872
	0.82	0.884	0.887	0.889	0.897	0.839	0.851	0.881	0.892	0.831	0.824	0.809	0.816	0.820	0.844	0.577	0.532	0.859
	0.84	0.851	0.853	0.846	0.858	0.797	0.811	0.848	0.866	0.816	0.805	0.786	0.795	0.801	0.827	0.501	0.460	0.843
	0.86	0.819	0.823	0.797	0.813	0.736	0.750	0.790	0.808	0.783	0.769	0.742	0.757	0.765	0.798	0.404	0.367	0.806
	0.88	0.739	0.747	0.715	0.733	0.657	0.672	0.711	0.733	0.717	0.702	0.668	0.684	0.699	0.743	0.315	0.290	0.742
	0.90	0.626	0.631	0.607	0.625	0.553	0.567	0.609	0.632	0.632	0.614	0.572	0.588	0.610	0.660	0.236	0.225	0.651
	0.92	0.514	0.519	0.472	0.491	0.439	0.453	0.478	0.503	0.487	0.472	0.431	0.443	0.468	0.521	0.156	0.145	0.503
	0.94	0.328	0.336	0.332	0.350	0.313	0.322	0.337	0.358	0.353	0.337	0.303	0.311	0.335	0.388	0.100	0.103	0.362
	0.96	0.237	0.240	0.202	0.208	0.205	0.210	0.222	0.231	0.211	0.202	0.181	0.180	0.201	0.242	0.060	0.070	0.217
	0.98	0.097	0.098	0.103	0.107	0.114	0.117	0.118	0.121	0.110	0.104	0.093	0.090	0.103	0.124	0.033	0.052	0.112
200	0.80	0.999	0.999	0.997	0.997	0.986	0.987	0.996	0.996	0.952	0.949	0.944	0.947	0.948	0.970	0.887	0.842	0.964
	0.82	0.996	0.996	0.995	0.995	0.985	0.986	0.995	0.996	0.954	0.950	0.944	0.949	0.949	0.969	0.881	0.832	0.964
	0.84	0.996	0.996	0.994	0.996	0.984	0.985	0.992	0.994	0.951	0.949	0.942	0.946	0.947	0.963	0.876	0.823	0.964
	0.86	0.994	0.994	0.988	0.990	0.980	0.983	0.988	0.989	0.947	0.943	0.937	0.940	0.943	0.956	0.859	0.790	0.957
	0.88	0.988	0.988	0.983	0.986	0.970	0.974	0.978	0.981	0.941	0.937	0.931	0.933	0.934	0.947	0.828	0.741	0.953
	0.90	0.970	0.971	0.963	0.968	0.943	0.950	0.966	0.970	0.930	0.924	0.916	0.919	0.922	0.929	0.758	0.647	0.944
	0.92	0.903	0.905	0.912	0.923	0.886	0.899	0.916	0.922	0.897	0.888	0.876	0.877	0.882	0.895	0.613	0.496	0.908
	0.94	0.810	0.811	0.767	0.791	0.735	0.757	0.777	0.792	0.793	0.778	0.745	0.755	0.772	0.791	0.410	0.310	0.805
	0.96	0.545	0.548	0.502	0.524	0.475	0.500	0.521	0.540	0.523	0.507	0.465	0.480	0.504	0.529	0.209	0.154	0.529
	0.98	0.223	0.224	0.206	0.213	0.207	0.217	0.224	0.229	0.220	0.208	0.190	0.192	0.207	0.221	0.085	0.071	0.219
400	0.80	1.000	1.000	1.000	1.000	0.999	0.999	1.000	1.000	1.000	0.999	0.999	0.999	0.999	1.000	0.991	0.987	1.000
	0.82	1.000	1.000	1.000	1.000	0.999	0.999	1.000	1.000	1.000	0.999	0.999	0.999	0.999	1.000	0.990	0.983	1.000
	0.84	1.000	1.000	1.000	1.000	0.999	0.999	1.000	1.000	1.000	0.999	0.999	0.999	0.999	1.000	0.986	0.975	1.000
	0.86	1.000	1.000	1.000	1.000	0.999	0.999	1.000	1.000	0.999	0.998	0.998	0.997	0.997	0.999	0.985	0.970	0.999
	0.88	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.997	0.998	0.997	0.997	0.999	0.980	0.956	0.999
	0.90	0.999	0.999	1.000	1.000	0.999	0.999	1.000	1.000	0.997	0.996	0.997	0.995	0.995	0.998	0.974	0.945	0.998
	0.92	0.999	1.000	0.999	1.000	0.999	0.999	0.999	0.999	0.995	0.993	0.993	0.992	0.992	0.995	0.963	0.918	0.996
	0.94	0.994	0.996	0.997	0.998	0.993	0.993	0.996	0.997	0.988	0.985	0.984	0.982	0.984	0.985	0.915	0.829	0.990
	0.96	0.889	0.936	0.950	0.962	0.936	0.942	0.949	0.958	0.949	0.938	0.931	0.931	0.935	0.940	0.700	0.547	0.952
	0.98	0.415	0.489	0.544	0.575	0.516	0.529	0.543	0.568	0.559	0.542	0.507	0.527	0.539	0.551	0.258	0.170	0.560

Table 4. Power of the tests for model with an intercept and a trend, $\phi = 0$

N	ρ	BNM_0	$BEPO_0$	BNM_b	$BEPO_b$	BNM_a	$BEPO_a$	BNM_h	$BEPO_h$	MZ_a	MZ_t	MSB	P_t	MP_t	ADF	MZ_{aLS}	ADF_{LS}	MZ_{a2}
50	0.80	0.272	0.275	0.236	0.234	0.173	0.173	0.213	0.213	0.092	0.095	0.095	0.106	0.101	0.468	0.009	0.262	0.082
	0.82	0.217	0.218	0.198	0.196	0.159	0.159	0.186	0.186	0.075	0.077	0.074	0.089	0.082	0.416	0.007	0.238	0.066
	0.84	0.210	0.210	0.170	0.167	0.140	0.141	0.161	0.161	0.057	0.057	0.059	0.064	0.059	0.349	0.005	0.196	0.051
	0.86	0.151	0.153	0.139	0.136	0.120	0.119	0.134	0.134	0.046	0.047	0.048	0.054	0.050	0.300	0.005	0.171	0.041
	0.88	0.115	0.116	0.111	0.110	0.110	0.109	0.112	0.113	0.037	0.038	0.036	0.042	0.040	0.250	0.003	0.144	0.031
	0.90	0.092	0.092	0.093	0.091	0.090	0.090	0.094	0.093	0.029	0.029	0.030	0.032	0.031	0.209	0.002	0.125	0.025
	0.92	0.092	0.093	0.082	0.081	0.080	0.080	0.087	0.086	0.020	0.020	0.022	0.024	0.022	0.172	0.002	0.106	0.019
	0.94	0.064	0.064	0.069	0.069	0.070	0.069	0.069	0.069	0.017	0.017	0.018	0.019	0.017	0.144	0.002	0.097	0.014
	0.96	0.045	0.045	0.050	0.049	0.067	0.066	0.066	0.066	0.012	0.013	0.013	0.015	0.014	0.117	0.001	0.082	0.010
	0.98	0.052	0.053	0.043	0.042	0.056	0.056	0.056	0.056	0.010	0.010	0.010	0.011	0.011	0.094	0.001	0.071	0.009
100	0.80	0.684	0.683	0.703	0.705	0.572	0.569	0.628	0.627	0.505	0.509	0.498	0.519	0.515	0.632	0.339	0.457	0.546
	0.82	0.623	0.615	0.633	0.635	0.508	0.506	0.554	0.553	0.429	0.433	0.423	0.438	0.437	0.573	0.262	0.392	0.458
	0.84	0.570	0.564	0.543	0.547	0.434	0.432	0.481	0.478	0.343	0.344	0.342	0.349	0.347	0.500	0.195	0.312	0.368
	0.86	0.478	0.471	0.458	0.461	0.364	0.361	0.395	0.393	0.260	0.261	0.259	0.265	0.266	0.409	0.140	0.245	0.279
	0.88	0.356	0.355	0.368	0.370	0.297	0.294	0.315	0.315	0.186	0.186	0.184	0.189	0.191	0.321	0.094	0.185	0.201
	0.90	0.281	0.273	0.278	0.281	0.220	0.218	0.230	0.228	0.133	0.135	0.132	0.137	0.139	0.250	0.069	0.146	0.145
	0.92	0.198	0.192	0.200	0.201	0.166	0.163	0.168	0.166	0.088	0.088	0.088	0.088	0.090	0.168	0.044	0.102	0.094
	0.94	0.123	0.121	0.131	0.132	0.122	0.120	0.111	0.110	0.054	0.055	0.054	0.054	0.056	0.115	0.027	0.077	0.057
	0.96	0.092	0.091	0.091	0.092	0.086	0.086	0.077	0.076	0.037	0.037	0.037	0.038	0.039	0.081	0.016	0.056	0.039
	0.98	0.074	0.072	0.062	0.062	0.065	0.064	0.051	0.051	0.023	0.024	0.025	0.024	0.025	0.052	0.013	0.046	0.025
200	0.80	0.964	0.964	0.968	0.968	0.947	0.948	0.967	0.965	0.844	0.844	0.844	0.845	0.845	0.845	0.844	0.800	0.890
	0.82	0.960	0.959	0.953	0.953	0.929	0.930	0.952	0.951	0.839	0.838	0.837	0.838	0.838	0.836	0.818	0.769	0.879
	0.84	0.926	0.926	0.933	0.933	0.902	0.903	0.928	0.926	0.829	0.828	0.829	0.830	0.829	0.829	0.789	0.748	0.866
	0.86	0.885	0.883	0.887	0.887	0.855	0.856	0.894	0.892	0.792	0.792	0.792	0.790	0.790	0.796	0.710	0.672	0.828
	0.88	0.849	0.848	0.824	0.824	0.784	0.786	0.827	0.823	0.724	0.723	0.721	0.723	0.725	0.747	0.609	0.583	0.757
	0.90	0.718	0.718	0.722	0.722	0.660	0.662	0.710	0.705	0.603	0.603	0.601	0.603	0.608	0.654	0.470	0.457	0.634
	0.92	0.566	0.564	0.570	0.570	0.514	0.516	0.567	0.562	0.431	0.431	0.432	0.431	0.436	0.501	0.306	0.312	0.452
	0.94	0.374	0.371	0.370	0.370	0.331	0.333	0.382	0.376	0.253	0.256	0.253	0.253	0.261	0.318	0.171	0.191	0.263
	0.96	0.183	0.182	0.193	0.193	0.185	0.186	0.198	0.195	0.119	0.120	0.117	0.119	0.123	0.158	0.078	0.099	0.124
	0.98	0.070	0.069	0.086	0.086	0.084	0.086	0.093	0.091	0.049	0.049	0.049	0.049	0.049	0.068	0.035	0.055	0.052
400	0.80	1.000	1.000	0.999	0.999	0.997	0.997	1.000	1.000	0.974	0.973	0.975	0.973	0.973	0.984	0.965	0.943	0.986
	0.82	1.000	1.000	0.999	0.999	0.997	0.997	0.998	0.998	0.972	0.971	0.972	0.970	0.970	0.982	0.964	0.936	0.984
	0.84	0.999	0.999	0.998	0.998	0.996	0.996	0.998	0.998	0.966	0.965	0.966	0.966	0.965	0.971	0.957	0.924	0.978
	0.86	0.996	0.996	0.996	0.996	0.991	0.991	0.995	0.995	0.963	0.963	0.964	0.963	0.963	0.966	0.953	0.917	0.975
	0.88	0.992	0.992	0.991	0.991	0.985	0.985	0.988	0.988	0.956	0.955	0.956	0.955	0.955	0.954	0.945	0.901	0.969
	0.90	0.980	0.980	0.977	0.977	0.971	0.971	0.975	0.975	0.948	0.947	0.948	0.947	0.947	0.942	0.931	0.883	0.963
	0.92	0.955	0.956	0.941	0.941	0.933	0.933	0.943	0.943	0.921	0.923	0.921	0.923	0.925	0.916	0.884	0.819	0.938
	0.94	0.851	0.851	0.844	0.845	0.825	0.824	0.849	0.847	0.816	0.817	0.813	0.816	0.819	0.822	0.709	0.633	0.834
	0.96	0.643	0.644	0.580	0.580	0.568	0.567	0.584	0.581	0.500	0.502	0.499	0.502	0.506	0.532	0.377	0.325	0.511
	0.98	0.228	0.228	0.199	0.199	0.191	0.191	0.211	0.208	0.145	0.146	0.146	0.146	0.148	0.163	0.107	0.102	0.147

Table 5. Size-adjusted power of the tests for model with an intercept only, $\phi = -0.5$

N	ρ	BNM_0	$BEPO_0$	BNM_b	$BEPO_b$	BNM_a	$BEPO_a$	BNM_h	$BEPO_h$	MZ_a	MZ_t	MSB	P_t	MP_t	ADF	MZ_{aLS}	ADF_{LS}	MZ_{a2}
50	0.80	0.389	0.394	0.411	0.417	0.307	0.308	0.397	0.392	0.657	0.670	0.606	0.689	0.605	0.652	0.555	0.430	0.707
	0.82	0.399	0.406	0.387	0.394	0.242	0.246	0.355	0.350	0.585	0.602	0.533	0.625	0.545	0.582	0.487	0.373	0.633
	0.84	0.34	0.348	0.335	0.344	0.234	0.234	0.332	0.327	0.496	0.514	0.448	0.553	0.473	0.493	0.408	0.312	0.539
	0.86	0.256	0.269	0.290	0.295	0.226	0.226	0.282	0.278	0.428	0.447	0.383	0.474	0.415	0.425	0.353	0.263	0.467
	0.88	0.221	0.226	0.257	0.262	0.179	0.179	0.262	0.260	0.353	0.371	0.309	0.399	0.347	0.352	0.291	0.216	0.379
	0.90	0.201	0.205	0.210	0.215	0.191	0.193	0.192	0.188	0.284	0.299	0.256	0.316	0.282	0.282	0.235	0.174	0.307
	0.92	0.139	0.144	0.157	0.163	0.141	0.143	0.168	0.164	0.224	0.234	0.197	0.245	0.219	0.223	0.195	0.144	0.231
	0.94	0.118	0.119	0.138	0.142	0.120	0.120	0.130	0.125	0.166	0.174	0.150	0.174	0.162	0.166	0.149	0.119	0.170
	0.96	0.093	0.1	0.110	0.114	0.101	0.099	0.087	0.086	0.110	0.114	0.103	0.115	0.108	0.110	0.105	0.092	0.111
	0.98	0.055	0.056	0.084	0.086	0.071	0.071	0.078	0.075	0.073	0.076	0.068	0.074	0.071	0.072	0.067	0.064	0.074
100	0.80	0.726	0.725	0.810	0.813	0.776	0.784	0.801	0.806	0.612	0.616	0.603	0.633	0.615	0.710	0.539	0.485	0.640
	0.82	0.695	0.693	0.768	0.774	0.741	0.749	0.754	0.762	0.598	0.599	0.591	0.615	0.598	0.690	0.512	0.443	0.625
	0.84	0.697	0.695	0.716	0.723	0.689	0.699	0.711	0.719	0.574	0.577	0.562	0.594	0.577	0.666	0.476	0.399	0.597
	0.86	0.64	0.637	0.643	0.649	0.639	0.647	0.647	0.654	0.551	0.553	0.536	0.569	0.551	0.626	0.429	0.344	0.566
	0.88	0.583	0.579	0.567	0.573	0.555	0.565	0.571	0.580	0.513	0.518	0.495	0.534	0.516	0.574	0.386	0.287	0.521
	0.90	0.508	0.501	0.483	0.490	0.475	0.483	0.487	0.494	0.444	0.450	0.435	0.466	0.450	0.500	0.333	0.239	0.459
	0.92	0.41	0.405	0.379	0.383	0.382	0.389	0.382	0.389	0.363	0.367	0.353	0.382	0.368	0.402	0.260	0.178	0.372
	0.94	0.299	0.297	0.281	0.283	0.278	0.286	0.277	0.283	0.281	0.285	0.270	0.297	0.286	0.309	0.201	0.127	0.285
	0.96	0.192	0.191	0.177	0.180	0.184	0.188	0.181	0.186	0.189	0.192	0.182	0.198	0.193	0.202	0.140	0.092	0.191
	0.98	0.115	0.111	0.108	0.109	0.104	0.105	0.101	0.104	0.109	0.110	0.107	0.111	0.112	0.113	0.099	0.070	0.112
200	0.80	0.914	0.914	0.985	0.986	0.968	0.971	0.978	0.978	0.864	0.864	0.855	0.873	0.864	0.969	0.792	0.772	0.884
	0.82	0.911	0.911	0.982	0.984	0.961	0.965	0.976	0.976	0.865	0.866	0.853	0.875	0.867	0.963	0.779	0.752	0.882
	0.84	0.897	0.897	0.979	0.981	0.957	0.961	0.971	0.972	0.856	0.857	0.844	0.866	0.858	0.947	0.773	0.726	0.875
	0.86	0.886	0.886	0.973	0.976	0.949	0.956	0.966	0.966	0.845	0.844	0.837	0.851	0.843	0.929	0.747	0.682	0.863
	0.88	0.872	0.873	0.955	0.959	0.927	0.934	0.947	0.951	0.836	0.834	0.825	0.840	0.833	0.909	0.721	0.628	0.852
	0.90	0.831	0.832	0.933	0.940	0.891	0.901	0.915	0.917	0.812	0.810	0.799	0.819	0.810	0.882	0.673	0.555	0.825
	0.92	0.788	0.789	0.862	0.875	0.823	0.838	0.852	0.857	0.771	0.769	0.753	0.780	0.769	0.828	0.603	0.446	0.778
	0.94	0.677	0.68	0.727	0.744	0.690	0.711	0.707	0.716	0.675	0.677	0.653	0.687	0.678	0.723	0.473	0.306	0.677
	0.96	0.501	0.504	0.489	0.511	0.460	0.483	0.476	0.483	0.467	0.468	0.441	0.477	0.472	0.499	0.306	0.172	0.468
	0.98	0.237	0.239	0.217	0.225	0.205	0.216	0.203	0.206	0.217	0.218	0.208	0.221	0.219	0.227	0.160	0.089	0.217
400	0.80	0.99	0.99	0.999	1.000	0.994	0.994	0.998	0.998	0.994	0.993	0.993	0.992	0.992	1.000	0.974	0.992	0.995
	0.82	0.989	0.989	1.000	1.000	0.996	0.996	0.999	0.999	0.994	0.994	0.994	0.994	0.993	1.000	0.974	0.988	0.996
	0.84	0.987	0.987	1.000	1.000	0.996	0.996	0.999	0.999	0.993	0.992	0.993	0.992	0.992	1.000	0.971	0.981	0.995
	0.86	0.987	0.987	1.000	1.000	0.996	0.997	0.999	0.999	0.993	0.992	0.992	0.991	0.991	1.000	0.971	0.973	0.995
	0.88	0.982	0.981	1.000	1.000	0.996	0.997	0.999	0.999	0.990	0.988	0.989	0.988	0.987	0.998	0.961	0.950	0.991
	0.90	0.978	0.981	0.999	0.999	0.996	0.996	0.999	0.999	0.989	0.986	0.989	0.985	0.985	0.998	0.955	0.926	0.991
	0.92	0.962	0.963	0.998	0.998	0.994	0.995	0.996	0.996	0.981	0.979	0.980	0.976	0.976	0.992	0.934	0.871	0.984
	0.94	0.942	0.942	0.990	0.990	0.984	0.988	0.989	0.990	0.963	0.959	0.961	0.957	0.955	0.976	0.882	0.761	0.968
	0.96	0.851	0.854	0.925	0.933	0.910	0.921	0.922	0.932	0.894	0.890	0.885	0.886	0.885	0.906	0.727	0.514	0.900
	0.98	0.504	0.502	0.522	0.533	0.507	0.528	0.507	0.527	0.519	0.519	0.497	0.519	0.521	0.527	0.354	0.186	0.525

Table 6. Power of the tests for model Constant and Trend, $\phi = -0.5$

N				SWZ	SWZ	AKA	AKA	HQ	HQ									
N	ρ	BNM_0	$BEPO_0$	BNM_b	$BEPO_b$	BNM_a	$BEPO_a$	BNM_h	$BEPO_h$	MZ_a	MZ_t	MSB	P_t	MP_t	ADF	MZ_{aLS}	ADF_{LS}	MZ_{a2}
50	0.80	0.211	0.209	0.211	0.212	0.151	0.151	0.159	0.172	0.285	0.277	0.265	0.293	0.266	0.274	0.291	0.207	0.313
	0.82	0.196	0.193	0.189	0.192	0.128	0.128	0.155	0.165	0.239	0.235	0.227	0.248	0.230	0.232	0.245	0.181	0.262
	0.84	0.169	0.166	0.164	0.165	0.113	0.114	0.130	0.139	0.204	0.203	0.190	0.207	0.193	0.196	0.206	0.156	0.221
	0.86	0.124	0.120	0.134	0.135	0.105	0.104	0.114	0.122	0.173	0.171	0.160	0.173	0.166	0.170	0.173	0.135	0.179
	0.88	0.097	0.097	0.116	0.117	0.083	0.084	0.099	0.105	0.137	0.136	0.130	0.139	0.135	0.134	0.139	0.110	0.143
	0.90	0.081	0.076	0.097	0.098	0.075	0.076	0.081	0.087	0.112	0.112	0.105	0.110	0.113	0.110	0.110	0.098	0.112
	0.92	0.074	0.073	0.084	0.086	0.070	0.070	0.073	0.078	0.089	0.089	0.088	0.088	0.088	0.091	0.090	0.081	0.090
	0.94	0.074	0.074	0.071	0.072	0.063	0.063	0.055	0.060	0.073	0.072	0.073	0.070	0.072	0.072	0.074	0.067	0.074
	0.96	0.062	0.061	0.059	0.059	0.054	0.054	0.052	0.056	0.061	0.059	0.060	0.059	0.059	0.060	0.061	0.058	0.060
	0.98	0.053	0.051	0.053	0.053	0.051	0.051	0.047	0.050	0.049	0.048	0.048	0.045	0.048	0.049	0.048	0.050	0.050
100	0.80	0.603	0.585	0.583	0.586	0.489	0.496	0.570	0.568	0.431	0.436	0.428	0.444	0.438	0.470	0.416	0.386	0.461
	0.82	0.552	0.541	0.516	0.519	0.432	0.439	0.497	0.496	0.397	0.400	0.394	0.406	0.400	0.429	0.376	0.346	0.424
	0.84	0.532	0.517	0.458	0.460	0.374	0.381	0.431	0.430	0.355	0.359	0.351	0.365	0.359	0.386	0.332	0.297	0.375
	0.86	0.445	0.432	0.386	0.388	0.308	0.313	0.365	0.364	0.308	0.312	0.307	0.318	0.313	0.337	0.276	0.245	0.313
	0.88	0.365	0.348	0.300	0.303	0.258	0.265	0.302	0.301	0.262	0.266	0.260	0.271	0.266	0.279	0.235	0.204	0.267
	0.90	0.296	0.284	0.241	0.243	0.205	0.210	0.227	0.227	0.219	0.220	0.217	0.225	0.222	0.229	0.192	0.172	0.219
	0.92	0.218	0.208	0.170	0.171	0.150	0.155	0.181	0.180	0.161	0.163	0.162	0.164	0.164	0.169	0.150	0.130	0.166
	0.94	0.138	0.129	0.122	0.123	0.123	0.127	0.127	0.126	0.117	0.119	0.115	0.119	0.119	0.121	0.112	0.098	0.121
	0.96	0.104	0.102	0.079	0.080	0.085	0.087	0.094	0.093	0.083	0.083	0.084	0.084	0.083	0.084	0.084	0.075	0.087
	0.98	0.081	0.076	0.062	0.062	0.059	0.060	0.066	0.065	0.066	0.066	0.066	0.067	0.067	0.068	0.066	0.061	0.067
200	0.80	0.873	0.873	0.923	0.926	0.711	0.901	0.930	0.930	0.685	0.683	0.684	0.687	0.682	0.760	0.691	0.684	0.751
	0.82	0.843	0.843	0.907	0.909	0.650	0.877	0.908	0.910	0.673	0.670	0.672	0.676	0.668	0.745	0.660	0.646	0.727
	0.84	0.834	0.836	0.871	0.875	0.578	0.844	0.877	0.878	0.663	0.661	0.663	0.665	0.659	0.723	0.639	0.618	0.714
	0.86	0.767	0.770	0.815	0.821	0.492	0.796	0.830	0.832	0.628	0.626	0.625	0.630	0.624	0.686	0.587	0.551	0.670
	0.88	0.701	0.703	0.742	0.748	0.388	0.719	0.756	0.760	0.588	0.585	0.586	0.590	0.583	0.636	0.527	0.481	0.622
	0.90	0.621	0.625	0.642	0.649	0.286	0.625	0.668	0.672	0.523	0.521	0.521	0.528	0.520	0.563	0.451	0.397	0.548
	0.92	0.465	0.468	0.498	0.507	0.183	0.494	0.524	0.530	0.431	0.429	0.428	0.434	0.427	0.465	0.353	0.298	0.447
	0.94	0.308	0.310	0.333	0.342	0.096	0.343	0.358	0.363	0.298	0.296	0.298	0.300	0.296	0.317	0.245	0.197	0.307
	0.96	0.186	0.187	0.187	0.191	0.050	0.184	0.203	0.206	0.170	0.168	0.168	0.171	0.168	0.179	0.137	0.114	0.169
	0.98	0.096	0.097	0.091	0.094	0.020	0.090	0.094	0.096	0.089	0.088	0.088	0.087	0.086	0.090	0.078	0.069	0.089
400	0.80	0.991	0.991	0.998	0.998	0.991	0.991	0.996	0.996	0.920	0.919	0.920	0.919	0.917	0.990	0.921	0.952	0.951
	0.82	0.986	0.986	0.997	0.997	0.990	0.990	0.995	0.995	0.923	0.923	0.921	0.923	0.920	0.987	0.915	0.940	0.949
	0.84	0.987	0.986	0.995	0.995	0.989	0.990	0.992	0.993	0.917	0.915	0.916	0.917	0.913	0.980	0.906	0.917	0.943
	0.86	0.980	0.979	0.990	0.990	0.985	0.985	0.986	0.986	0.918	0.916	0.917	0.917	0.914	0.973	0.903	0.897	0.941
	0.88	0.965	0.964	0.983	0.982	0.972	0.973	0.979	0.979	0.902	0.901	0.899	0.902	0.899	0.951	0.879	0.859	0.923
	0.90	0.949	0.946	0.962	0.961	0.955	0.956	0.961	0.962	0.888	0.887	0.886	0.889	0.885	0.928	0.853	0.813	0.909
	0.92	0.900	0.899	0.930	0.929	0.905	0.907	0.910	0.912	0.847	0.846	0.843	0.847	0.842	0.879	0.792	0.727	0.865
	0.94	0.812	0.811	0.814	0.812	0.803	0.807	0.801	0.804	0.747	0.748	0.740	0.751	0.745	0.775	0.659	0.566	0.758
	0.96	0.589	0.582	0.562	0.559	0.559	0.564	0.540	0.545	0.513	0.514	0.501	0.519	0.512	0.529	0.421	0.325	0.515
	0.98	0.228	0.218	0.208	0.207	0.205	0.208	0.198	0.201	0.198	0.197	0.191	0.200	0.196	0.198	0.158	0.119	0.191

Table 7. Power of the tests for model with an intercept only, $\phi = 0$

N	ρ	BNM_0	$BEPO_0$	BNM_b	$BEPO_b$	BNM_a	$BEPO_a$	BNM_h	$BEPO_h$	MZ_a	MZ_t	MSB	P_t	MP_t	ADF	MZ_{aLS}	ADF_{LS}	MZ_{a2}
50	0.1	0.959	0.955	0.950	0.952	0.773	0.779	0.868	0.880	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.2	0.950	0.943	0.925	0.929	0.772	0.776	0.871	0.878	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.3	0.934	0.927	0.918	0.919	0.746	0.753	0.848	0.855	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000
	0.4	0.917	0.907	0.894	0.898	0.725	0.731	0.824	0.833	1.000	1.000	1.000	1.000	1.000	1.000	0.994	0.999	1.000
	0.5	0.896	0.886	0.861	0.864	0.693	0.699	0.795	0.806	1.000	1.000	1.000	0.999	0.999	1.000	0.954	0.987	1.000
	0.6	0.854	0.840	0.817	0.826	0.610	0.619	0.727	0.741	0.997	0.996	0.994	0.993	0.994	0.999	0.810	0.921	0.997
	0.7	0.783	0.763	0.741	0.748	0.497	0.508	0.624	0.638	0.965	0.955	0.942	0.951	0.950	0.984	0.513	0.699	0.965
	0.8	0.550	0.566	0.530	0.542	0.336	0.338	0.408	0.424	0.772	0.753	0.713	0.737	0.748	0.862	0.216	0.383	0.769
	0.9	0.211	0.222	0.241	0.244	0.179	0.179	0.188	0.196	0.366	0.346	0.319	0.309	0.345	0.488	0.061	0.141	0.355
	1	0.054	0.044	0.053	0.053	0.047	0.047	0.047	0.047	0.062	0.056	0.058	0.044	0.056	0.096	0.008	0.057	0.057
100	0.1	0.999	0.998	0.991	0.992	0.941	0.947	0.975	0.978	0.715	0.715	0.702	0.716	0.714	0.893	0.698	0.742	0.779
	0.2	0.996	0.996	0.992	0.992	0.949	0.954	0.983	0.984	0.742	0.739	0.730	0.741	0.739	0.886	0.711	0.743	0.797
	0.3	0.994	0.994	0.990	0.991	0.950	0.951	0.981	0.983	0.756	0.754	0.744	0.756	0.753	0.883	0.725	0.744	0.811
	0.4	0.992	0.992	0.989	0.990	0.949	0.952	0.979	0.983	0.784	0.783	0.772	0.785	0.783	0.879	0.739	0.743	0.832
	0.5	0.987	0.988	0.983	0.985	0.955	0.958	0.977	0.979	0.802	0.800	0.790	0.801	0.799	0.874	0.755	0.744	0.850
	0.6	0.982	0.981	0.976	0.978	0.953	0.956	0.969	0.974	0.823	0.820	0.809	0.819	0.818	0.874	0.765	0.741	0.858
	0.7	0.966	0.965	0.961	0.965	0.928	0.933	0.957	0.963	0.834	0.830	0.820	0.830	0.830	0.868	0.749	0.710	0.868
	0.8	0.926	0.930	0.913	0.922	0.874	0.882	0.901	0.911	0.841	0.836	0.823	0.831	0.834	0.855	0.646	0.597	0.872
	0.9	0.626	0.631	0.607	0.625	0.553	0.567	0.609	0.632	0.632	0.614	0.572	0.588	0.610	0.660	0.236	0.225	0.651
	1	0.051	0.047	0.048	0.047	0.051	0.052	0.052	0.054	0.047	0.044	0.044	0.038	0.043	0.053	0.015	0.042	0.048
200	0.1	1.000	1.000	0.999	0.999	0.970	0.975	0.991	0.992	0.859	0.855	0.842	0.855	0.854	0.998	0.795	0.920	0.896
	0.2	1.000	1.000	0.998	0.999	0.978	0.980	0.993	0.993	0.882	0.880	0.868	0.881	0.880	0.997	0.819	0.910	0.911
	0.3	1.000	1.000	0.999	0.999	0.977	0.980	0.998	0.998	0.898	0.895	0.887	0.896	0.894	0.996	0.832	0.899	0.924
	0.4	1.000	1.000	1.000	1.000	0.978	0.981	0.996	0.996	0.918	0.915	0.906	0.914	0.913	0.995	0.844	0.893	0.939
	0.5	1.000	1.000	1.000	1.000	0.980	0.983	0.998	0.998	0.930	0.927	0.920	0.926	0.925	0.993	0.860	0.878	0.949
	0.6	1.000	1.000	0.999	0.999	0.986	0.987	0.997	0.997	0.946	0.941	0.935	0.941	0.941	0.991	0.873	0.867	0.959
	0.7	0.999	0.999	0.999	0.999	0.989	0.990	0.997	0.997	0.953	0.950	0.945	0.949	0.948	0.987	0.885	0.862	0.964
	0.8	0.999	0.999	0.997	0.997	0.986	0.987	0.996	0.996	0.952	0.949	0.944	0.947	0.948	0.970	0.887	0.842	0.964
	0.9	0.967	0.969	0.963	0.969	0.940	0.947	0.963	0.970	0.926	0.921	0.911	0.915	0.919	0.927	0.754	0.644	0.939
	1	0.050	0.047	0.048	0.046	0.048	0.050	0.056	0.055	0.051	0.046	0.046	0.042	0.046	0.050	0.022	0.046	0.052
400	0.1	1.000	1.000	1.000	1.000	0.985	0.986	0.998	0.998	0.982	0.981	0.979	0.979	0.979	1.000	0.942	1.000	0.987
	0.2	1.000	1.000	1.000	1.000	0.988	0.990	0.999	0.999	0.990	0.989	0.988	0.989	0.989	1.000	0.958	1.000	0.994
	0.3	1.000	1.000	1.000	1.000	0.990	0.991	0.998	0.999	0.994	0.992	0.992	0.991	0.991	1.000	0.969	1.000	0.995
	0.4	1.000	1.000	1.000	1.000	0.993	0.993	0.998	0.998	0.996	0.996	0.996	0.995	0.995	1.000	0.980	1.000	0.998
	0.5	1.000	1.000	1.000	1.000	0.992	0.993	0.999	0.999	0.998	0.998	0.998	0.997	0.997	1.000	0.983	0.999	0.999
	0.6	1.000	1.000	1.000	1.000	0.994	0.994	1.000	1.000	1.000	0.999	1.000	0.999	0.999	1.000	0.990	1.000	1.000
	0.7	1.000	1.000	1.000	1.000	0.998	0.997	1.000	1.000	1.000	1.000	0.999	0.999	0.999	1.000	0.992	0.997	1.000
	0.8	1.000	1.000	1.000	1.000	0.999	0.999	1.000	1.000	1.000	0.999	0.999	0.999	0.999	1.000	0.991	0.987	1.000
	0.9	0.999	0.999	1.000	1.000	0.999	0.999	1.000	1.000	0.997	0.996	0.997	0.995	0.995	0.998	0.974	0.945	0.998
	1	0.047	0.045	0.054	0.055	0.043	0.041	0.048	0.046	0.047	0.043	0.043	0.040	0.043	0.044	0.025	0.042	0.047

Table 8. Power of the tests for model with an intercept and a trend, $\phi = 0$

N	ρ	BNM_0	$BEPO_0$	BNM_b	$BEPO_b$	BNM_a	$BEPO_a$	BNM_h	$BEPO_h$	MZ_a	MZ_t	MSB	P_t	MP_t	ADF	MZ_{aLS}	ADF_{LS}	MZ_{a2}
50	0.1	0.972	0.957	0.924	0.923	0.661	0.669	0.829	0.830	1.000	1.000	1.000	0.999	0.999	1.000	0.985	1.000	1.000
	0.2	0.953	0.929	0.906	0.905	0.651	0.661	0.802	0.802	0.999	0.998	0.999	0.997	0.996	1.000	0.941	1.000	0.999
	0.3	0.929	0.895	0.873	0.871	0.626	0.639	0.794	0.795	0.990	0.986	0.989	0.988	0.983	1.000	0.808	0.999	0.989
	0.4	0.895	0.846	0.823	0.820	0.557	0.572	0.734	0.734	0.948	0.944	0.944	0.956	0.940	0.999	0.574	0.990	0.944
	0.5	0.827	0.759	0.753	0.750	0.501	0.517	0.663	0.663	0.804	0.803	0.800	0.833	0.801	0.992	0.308	0.938	0.794
	0.6	0.719	0.626	0.624	0.620	0.390	0.406	0.548	0.548	0.544	0.544	0.540	0.598	0.556	0.946	0.130	0.786	0.526
	0.7	0.533	0.424	0.429	0.424	0.266	0.278	0.371	0.370	0.261	0.265	0.258	0.299	0.277	0.771	0.038	0.519	0.243
	0.8	0.272	0.275	0.236	0.234	0.173	0.173	0.213	0.213	0.092	0.095	0.095	0.106	0.101	0.468	0.009	0.262	0.082
	0.9	0.092	0.092	0.093	0.091	0.090	0.090	0.094	0.093	0.029	0.029	0.030	0.032	0.031	0.209	0.002	0.125	0.025
	1	0.055	0.035	0.048	0.047	0.053	0.054	0.051	0.051	0.010	0.010	0.010	0.011	0.011	0.090	0.001	0.068	0.009
100	0.1	0.999	0.999	0.994	0.994	0.866	0.865	0.959	0.959	0.605	0.606	0.604	0.610	0.607	0.687	0.723	0.760	0.759
	0.2	0.999	0.998	0.994	0.994	0.876	0.875	0.963	0.962	0.642	0.641	0.642	0.646	0.642	0.710	0.729	0.756	0.769
	0.3	0.997	0.996	0.991	0.991	0.867	0.866	0.964	0.963	0.670	0.670	0.668	0.675	0.671	0.722	0.742	0.759	0.785
	0.4	0.994	0.992	0.987	0.987	0.877	0.877	0.954	0.954	0.697	0.696	0.696	0.702	0.697	0.738	0.745	0.751	0.791
	0.5	0.983	0.978	0.978	0.978	0.867	0.866	0.948	0.947	0.720	0.720	0.719	0.725	0.721	0.748	0.750	0.743	0.800
	0.6	0.968	0.959	0.956	0.956	0.847	0.846	0.918	0.918	0.736	0.737	0.734	0.741	0.738	0.759	0.739	0.732	0.803
	0.7	0.911	0.889	0.896	0.895	0.788	0.786	0.847	0.846	0.700	0.700	0.698	0.705	0.701	0.735	0.635	0.671	0.760
	0.8	0.684	0.683	0.703	0.705	0.572	0.569	0.628	0.627	0.505	0.509	0.498	0.519	0.515	0.632	0.339	0.457	0.546
	0.9	0.281	0.273	0.278	0.281	0.220	0.218	0.230	0.228	0.133	0.135	0.132	0.137	0.139	0.250	0.069	0.146	0.145
	1	0.052	0.042	0.051	0.051	0.055	0.054	0.050	0.049	0.020	0.021	0.020	0.021	0.022	0.043	0.010	0.042	0.021
200	0.1	1.000	1.000	0.998	0.998	0.945	0.946	0.988	0.988	0.634	0.636	0.633	0.640	0.638	0.827	0.791	0.847	0.820
	0.2	1.000	1.000	0.998	0.998	0.949	0.949	0.992	0.992	0.684	0.685	0.683	0.688	0.686	0.840	0.811	0.854	0.844
	0.3	1.000	1.000	0.999	0.999	0.952	0.952	0.992	0.992	0.726	0.725	0.725	0.728	0.727	0.844	0.825	0.847	0.856
	0.4	1.000	1.000	1.000	1.000	0.955	0.955	0.991	0.991	0.762	0.762	0.762	0.763	0.762	0.849	0.830	0.840	0.863
	0.5	1.000	1.000	1.000	1.000	0.956	0.957	0.993	0.993	0.791	0.791	0.790	0.793	0.791	0.847	0.843	0.834	0.876
	0.6	0.999	0.999	0.999	0.999	0.967	0.967	0.996	0.996	0.814	0.813	0.814	0.814	0.813	0.848	0.852	0.834	0.888
	0.7	0.995	0.995	0.996	0.996	0.974	0.974	0.994	0.994	0.838	0.839	0.837	0.840	0.839	0.851	0.859	0.829	0.893
	0.8	0.964	0.964	0.968	0.968	0.947	0.948	0.967	0.965	0.844	0.844	0.844	0.845	0.845	0.845	0.844	0.800	0.890
	0.9	0.718	0.718	0.722	0.722	0.660	0.662	0.710	0.705	0.603	0.603	0.601	0.603	0.608	0.654	0.470	0.457	0.634
	1	0.050	0.044	0.056	0.056	0.047	0.048	0.049	0.048	0.027	0.027	0.028	0.027	0.028	0.039	0.020	0.042	0.028
400	0.1	1.000	1.000	0.999	0.999	0.972	0.972	0.997	0.997	0.785	0.785	0.785	0.786	0.785	0.991	0.908	0.997	0.937
	0.2	1.000	1.000	1.000	1.000	0.976	0.975	0.998	0.998	0.836	0.835	0.837	0.835	0.835	0.995	0.929	0.998	0.953
	0.3	1.000	1.000	1.000	1.000	0.977	0.977	0.997	0.997	0.879	0.877	0.881	0.877	0.876	0.996	0.937	0.994	0.963
	0.4	1.000	1.000	1.000	1.000	0.983	0.982	0.998	0.998	0.918	0.916	0.921	0.916	0.916	0.998	0.954	0.994	0.973
	0.5	1.000	1.000	1.000	1.000	0.984	0.983	0.999	0.999	0.941	0.939	0.942	0.939	0.938	0.998	0.957	0.990	0.976
	0.6	1.000	1.000	1.000	1.000	0.990	0.990	0.999	0.999	0.966	0.966	0.966	0.966	0.965	0.998	0.969	0.986	0.985
	0.7	1.000	1.000	1.000	1.000	0.996	0.996	1.000	1.000	0.974	0.974	0.975	0.974	0.973	0.995	0.971	0.971	0.986
	0.8	1.000	1.000	0.999	0.999	0.997	0.997	1.000	1.000	0.974	0.973	0.975	0.973	0.973	0.984	0.965	0.943	0.986
	0.9	0.980	0.980	0.977	0.977	0.971	0.971	0.975	0.975	0.948	0.947	0.948	0.947	0.947	0.942	0.931	0.883	0.963
	1	0.047	0.044	0.048	0.046	0.050	0.049	0.047	0.047	0.036	0.035	0.037	0.034	0.036	0.040	0.028	0.043	0.036

Table 9a, Relative frequencies of MA order chosen when only an intercept is included in the model

		$N = 50$						$N = 100$					
ϕ		0	1	2	3	4	5	0	1	2	3	4	5
−0.8	BIC	0.686	0.249	0.043	0.019	0.004	0.001	0.223	0.731	0.033	0.008	0.003	0.002
	AIC	0.394	0.278	0.110	0.087	0.070	0.062	0.094	0.603	0.116	0.072	0.059	0.057
	HQIC	0.527	0.291	0.082	0.054	0.028	0.019	0.152	0.713	0.076	0.029	0.019	0.013
−0.7	BIC	0.497	0.416	0.057	0.019	0.011	0.001	0.093	0.863	0.031	0.008	0.004	0.001
	AIC	0.271	0.366	0.117	0.085	0.092	0.071	0.031	0.673	0.112	0.072	0.056	0.055
	HQIC	0.378	0.406	0.099	0.055	0.044	0.020	0.055	0.814	0.071	0.030	0.017	0.012
−0.6	BIC	0.423	0.485	0.058	0.022	0.008	0.005	0.050	0.904	0.034	0.009	0.003	0.001
	AIC	0.199	0.455	0.101	0.088	0.086	0.072	0.014	0.681	0.121	0.071	0.056	0.056
	HQIC	0.296	0.501	0.086	0.055	0.042	0.021	0.027	0.834	0.076	0.033	0.018	0.012
−0.5	BIC	0.451	0.458	0.058	0.028	0.005	0.002	0.070	0.889	0.031	0.007	0.002	0.001
	AIC	0.207	0.432	0.112	0.098	0.083	0.069	0.014	0.700	0.116	0.069	0.048	0.053
	HQIC	0.311	0.485	0.090	0.060	0.035	0.021	0.032	0.843	0.072	0.029	0.013	0.012
−0.4	BIC	0.591	0.333	0.045	0.021	0.008	0.003	0.164	0.792	0.035	0.007	0.001	0.001
	AIC	0.298	0.353	0.109	0.091	0.082	0.069	0.039	0.667	0.123	0.070	0.050	0.051
	HQIC	0.438	0.384	0.086	0.047	0.033	0.015	0.082	0.786	0.077	0.032	0.014	0.009
−0.3	BIC	0.736	0.212	0.035	0.011	0.005	0.002	0.398	0.567	0.027	0.005	0.002	0.001
	AIC	0.410	0.283	0.110	0.071	0.058	0.070	0.132	0.592	0.108	0.064	0.053	0.051
	HQIC	0.557	0.283	0.071	0.040	0.028	0.023	0.240	0.647	0.064	0.023	0.015	0.010
−0.2	BIC	0.869	0.090	0.024	0.015	0.002	0.001	0.726	0.246	0.020	0.005	0.002	0.001
	AIC	0.561	0.176	0.075	0.069	0.062	0.058	0.365	0.381	0.102	0.062	0.044	0.046
	HQIC	0.717	0.150	0.051	0.042	0.024	0.018	0.546	0.353	0.057	0.023	0.012	0.009
−0.1	BIC	0.921	0.045	0.019	0.011	0.004	0.001	0.928	0.060	0.009	0.002	0.001	0.000
	AIC	0.666	0.091	0.063	0.061	0.065	0.056	0.641	0.166	0.068	0.047	0.039	0.039
	HQIC	0.814	0.072	0.045	0.031	0.026	0.013	0.821	0.115	0.033	0.014	0.010	0.008
0	BIC	0.920	0.055	0.014	0.007	0.005	0.001	0.959	0.031	0.006	0.002	0.001	0.000
	AIC	0.645	0.115	0.069	0.062	0.062	0.049	0.706	0.116	0.062	0.043	0.035	0.038
	HQIC	0.794	0.091	0.043	0.034	0.026	0.012	0.875	0.073	0.027	0.012	0.009	0.005
0.1	BIC	0.822	0.138	0.027	0.007	0.007	0.001	0.824	0.156	0.014	0.004	0.001	0.001
	AIC	0.525	0.208	0.086	0.057	0.063	0.063	0.492	0.289	0.085	0.053	0.042	0.040
	HQIC	0.672	0.188	0.058	0.035	0.034	0.014	0.676	0.247	0.044	0.019	0.009	0.006
0.2	BIC	0.624	0.317	0.037	0.013	0.009	0.001	0.500	0.468	0.025	0.005	0.002	0.001
	AIC	0.328	0.381	0.102	0.074	0.064	0.053	0.188	0.549	0.104	0.062	0.049	0.048
	HQIC	0.460	0.383	0.072	0.042	0.031	0.014	0.320	0.574	0.060	0.024	0.012	0.010
0.3	BIC	0.378	0.547	0.045	0.018	0.011	0.002	0.163	0.794	0.034	0.007	0.002	0.001
	AIC	0.140	0.517	0.114	0.087	0.072	0.072	0.034	0.672	0.121	0.070	0.054	0.050
	HQIC	0.241	0.579	0.086	0.044	0.034	0.018	0.076	0.795	0.079	0.026	0.016	0.010
0.4	BIC	0.167	0.754	0.047	0.021	0.011	0.002	0.025	0.932	0.033	0.007	0.002	0.002
	AIC	0.039	0.612	0.120	0.081	0.072	0.077	0.003	0.709	0.122	0.066	0.052	0.048
	HQIC	0.085	0.713	0.094	0.049	0.034	0.027	0.009	0.863	0.076	0.028	0.014	0.010
0.5	BIC	0.038	0.877	0.058	0.016	0.010	0.003	0.002	0.952	0.034	0.008	0.003	0.001
	AIC	0.009	0.650	0.127	0.084	0.074	0.058	0.000	0.710	0.117	0.074	0.053	0.046
	HQIC	0.018	0.777	0.103	0.049	0.037	0.017	0.000	0.866	0.074	0.031	0.018	0.010
0.6	BIC	0.006	0.912	0.055	0.019	0.007	0.003	0.000	0.954	0.034	0.007	0.003	0.002
	AIC	0.001	0.664	0.123	0.077	0.073	0.064	0.000	0.702	0.126	0.074	0.050	0.049
	HQIC	0.002	0.804	0.093	0.045	0.036	0.022	0.000	0.865	0.079	0.030	0.015	0.011
0.7	BIC	0.001	0.916	0.055	0.016	0.011	0.003	0.000	0.956	0.034	0.007	0.002	0.001
	AIC	0.000	0.682	0.117	0.068	0.072	0.062	0.000	0.712	0.126	0.067	0.046	0.048
	HQIC	0.000	0.816	0.094	0.039	0.035	0.017	0.000	0.873	0.077	0.028	0.013	0.009
0.8	BIC	0.001	0.912	0.059	0.021	0.007	0.002	0.000	0.957	0.034	0.006	0.002	0.001
	AIC	0.000	0.659	0.132	0.066	0.080	0.064	0.000	0.713	0.121	0.067	0.053	0.046
	HQIC	0.001	0.801	0.095	0.043	0.037	0.024	0.000	0.870	0.075	0.029	0.017	0.009

Table 9b. Relative frequencies of MA order chosen when only an intercept is included in the model

		$N = 200$						$N = 400$					
ϕ		0	1	2	3	4	5	0	1	2	3	4	5
-0.8	BIC	0.014	0.958	0.023	0.004	0.001	0.000	0.000	0.983	0.015	0.001	0.000	0.000
	AIC	0.003	0.721	0.121	0.069	0.045	0.042	0.000	0.730	0.123	0.065	0.048	0.035
	HQIC	0.007	0.893	0.062	0.023	0.009	0.006	0.000	0.918	0.056	0.017	0.006	0.002
-0.7	BIC	0.002	0.971	0.023	0.003	0.001	0.000	0.000	0.984	0.015	0.001	0.000	0.000
	AIC	0.001	0.722	0.123	0.068	0.047	0.039	0.000	0.744	0.117	0.064	0.040	0.036
	HQIC	0.001	0.898	0.066	0.022	0.010	0.004	0.000	0.924	0.053	0.016	0.005	0.002
-0.6	BIC	0.001	0.972	0.024	0.003	0.001	0.000	0.000	0.984	0.014	0.002	0.000	0.000
	AIC	0.000	0.724	0.122	0.067	0.047	0.040	0.000	0.734	0.125	0.068	0.041	0.033
	HQIC	0.000	0.897	0.067	0.022	0.010	0.004	0.000	0.917	0.056	0.018	0.006	0.004
-0.5	BIC	0.001	0.973	0.024	0.002	0.000	0.000	0.000	0.984	0.015	0.001	0.000	0.000
	AIC	0.000	0.734	0.119	0.064	0.045	0.038	0.000	0.737	0.121	0.064	0.041	0.037
	HQIC	0.000	0.906	0.062	0.019	0.009	0.003	0.000	0.923	0.055	0.015	0.005	0.002
-0.4	BIC	0.003	0.971	0.022	0.004	0.001	0.000	0.000	0.985	0.013	0.001	0.000	0.000
	AIC	0.000	0.731	0.119	0.065	0.045	0.039	0.000	0.741	0.124	0.062	0.041	0.032
	HQIC	0.001	0.901	0.063	0.020	0.010	0.005	0.000	0.925	0.055	0.014	0.004	0.003
-0.3	BIC	0.065	0.909	0.023	0.003	0.000	0.000	0.001	0.982	0.016	0.002	0.000	0.000
	AIC	0.007	0.725	0.123	0.066	0.043	0.037	0.000	0.739	0.122	0.063	0.040	0.036
	HQIC	0.023	0.881	0.064	0.020	0.008	0.004	0.000	0.917	0.059	0.016	0.006	0.003
-0.2	BIC	0.420	0.557	0.019	0.003	0.000	0.000	0.092	0.892	0.015	0.001	0.000	0.000
	AIC	0.111	0.628	0.115	0.064	0.046	0.036	0.007	0.732	0.115	0.064	0.045	0.037
	HQIC	0.237	0.673	0.059	0.018	0.008	0.005	0.029	0.894	0.054	0.015	0.006	0.003
-0.1	BIC	0.868	0.123	0.008	0.001	0.000	0.000	0.729	0.263	0.007	0.001	0.000	0.000
	AIC	0.503	0.290	0.085	0.051	0.040	0.031	0.286	0.486	0.101	0.057	0.040	0.029
	HQIC	0.716	0.232	0.032	0.013	0.006	0.002	0.512	0.433	0.038	0.011	0.005	0.001
0	BIC	0.975	0.022	0.003	0.001	0.000	0.000	0.983	0.016	0.002	0.000	0.000	0.000
	AIC	0.715	0.124	0.062	0.039	0.032	0.028	0.717	0.123	0.064	0.042	0.030	0.023
	HQIC	0.901	0.066	0.020	0.009	0.003	0.002	0.915	0.060	0.015	0.007	0.003	0.001
0.1	BIC	0.772	0.216	0.010	0.002	0.000	0.000	0.636	0.353	0.009	0.001	0.000	0.000
	AIC	0.369	0.408	0.093	0.055	0.039	0.036	0.204	0.560	0.107	0.056	0.043	0.031
	HQIC	0.583	0.354	0.041	0.013	0.006	0.004	0.406	0.532	0.044	0.012	0.004	0.002
0.2	BIC	0.258	0.716	0.022	0.003	0.000	0.000	0.051	0.935	0.012	0.001	0.000	0.000
	AIC	0.050	0.687	0.121	0.063	0.042	0.037	0.003	0.739	0.118	0.065	0.041	0.036
	HQIC	0.122	0.788	0.058	0.020	0.008	0.005	0.013	0.911	0.052	0.017	0.005	0.002
0.3	BIC	0.022	0.957	0.018	0.003	0.000	0.000	0.000	0.985	0.013	0.002	0.000	0.000
	AIC	0.001	0.731	0.119	0.066	0.046	0.036	0.000	0.745	0.119	0.063	0.040	0.034
	HQIC	0.005	0.900	0.063	0.018	0.011	0.004	0.000	0.921	0.056	0.016	0.005	0.002
0.4	BIC	0.000	0.974	0.023	0.003	0.000	0.000	0.000	0.984	0.014	0.002	0.000	0.000
	AIC	0.000	0.725	0.121	0.067	0.046	0.040	0.000	0.730	0.120	0.069	0.045	0.035
	HQIC	0.000	0.901	0.063	0.022	0.009	0.004	0.000	0.923	0.052	0.018	0.004	0.003
0.5	BIC	0.000	0.974	0.022	0.003	0.001	0.000	0.000	0.986	0.013	0.001	0.000	0.000
	AIC	0.000	0.727	0.124	0.064	0.047	0.038	0.000	0.741	0.116	0.064	0.044	0.036
	HQIC	0.000	0.903	0.064	0.022	0.008	0.004	0.000	0.927	0.051	0.013	0.006	0.003
0.6	BIC	0.000	0.972	0.024	0.003	0.001	0.000	0.000	0.984	0.015	0.001	0.000	0.000
	AIC	0.000	0.730	0.120	0.066	0.044	0.040	0.000	0.743	0.117	0.065	0.042	0.033
	HQIC	0.000	0.902	0.068	0.019	0.008	0.004	0.000	0.926	0.052	0.014	0.006	0.002
0.7	BIC	0.000	0.976	0.021	0.003	0.001	0.000	0.000	0.981	0.017	0.002	0.000	0.000
	AIC	0.000	0.727	0.121	0.066	0.047	0.039	0.000	0.739	0.119	0.066	0.041	0.035
	HQIC	0.000	0.904	0.065	0.020	0.009	0.003	0.000	0.920	0.058	0.015	0.005	0.002
0.8	BIC	0.000	0.973	0.023	0.003	0.001	0.000	0.000	0.983	0.016	0.001	0.000	0.000
	AIC	0.000	0.736	0.117	0.065	0.044	0.038	0.000	0.737	0.124	0.062	0.043	0.034
	HQIC	0.000	0.906	0.065	0.019	0.007	0.003	0.000	0.917	0.060	0.014	0.006	0.003

Table 10. Relative frequencies of information criteria when an intercept is included in the model

		$N=50$						$N=100$					
ρ		0	1	2	3	4	5	0	1	2	3	4	5
0.1	*BIC*	0.910	0.054	0.022	0.010	0.003	0.001	0.939	0.044	0.014	0.002	0.001	0.000
	AIC	0.605	0.126	0.095	0.070	0.058	0.045	0.615	0.135	0.094	0.061	0.047	0.048
	HQIC	0.777	0.099	0.058	0.036	0.020	0.010	0.824	0.092	0.047	0.021	0.010	0.007
0.2	*BIC*	0.915	0.049	0.023	0.009	0.003	0.001	0.946	0.040	0.012	0.001	0.001	0.000
	AIC	0.623	0.109	0.092	0.069	0.058	0.049	0.641	0.128	0.086	0.057	0.047	0.041
	HQIC	0.794	0.082	0.055	0.033	0.022	0.013	0.843	0.086	0.040	0.016	0.009	0.007
0.3	*BIC*	0.914	0.053	0.023	0.007	0.003	0.000	0.951	0.037	0.008	0.003	0.001	0.000
	AIC	0.626	0.112	0.087	0.063	0.064	0.049	0.663	0.113	0.076	0.058	0.048	0.042
	HQIC	0.788	0.092	0.056	0.032	0.020	0.012	0.855	0.073	0.035	0.020	0.011	0.006
0.4	*BIC*	0.908	0.059	0.021	0.008	0.003	0.001	0.948	0.039	0.009	0.002	0.000	0.000
	AIC	0.635	0.116	0.082	0.058	0.058	0.051	0.671	0.112	0.072	0.058	0.041	0.046
	HQIC	0.784	0.096	0.054	0.030	0.023	0.012	0.855	0.077	0.032	0.021	0.007	0.009
0.5	*BIC*	0.912	0.055	0.022	0.006	0.003	0.001	0.954	0.033	0.010	0.003	0.001	0.000
	AIC	0.643	0.115	0.074	0.058	0.061	0.049	0.681	0.112	0.075	0.050	0.046	0.036
	HQIC	0.791	0.094	0.054	0.027	0.023	0.011	0.860	0.074	0.034	0.016	0.009	0.006
0.6	*BIC*	0.909	0.055	0.025	0.006	0.004	0.001	0.954	0.034	0.008	0.002	0.001	0.000
	AIC	0.657	0.114	0.073	0.051	0.059	0.045	0.686	0.106	0.072	0.049	0.048	0.040
	HQIC	0.802	0.087	0.050	0.026	0.023	0.013	0.861	0.071	0.034	0.014	0.012	0.007
0.7	*BIC*	0.920	0.049	0.022	0.006	0.002	0.001	0.955	0.031	0.009	0.002	0.003	0.000
	AIC	0.658	0.110	0.077	0.062	0.053	0.041	0.690	0.108	0.066	0.049	0.045	0.041
	HQIC	0.808	0.082	0.051	0.027	0.022	0.011	0.868	0.072	0.031	0.013	0.010	0.007
0.8	*BIC*	0.913	0.052	0.022	0.007	0.004	0.001	0.959	0.031	0.006	0.003	0.001	0.000
	AIC	0.647	0.112	0.075	0.059	0.066	0.041	0.696	0.116	0.057	0.048	0.043	0.039
	HQIC	0.794	0.091	0.049	0.029	0.027	0.009	0.868	0.072	0.027	0.016	0.011	0.006
0.9	*BIC*	0.915	0.051	0.019	0.010	0.004	0.001	0.958	0.032	0.008	0.002	0.001	0.000
	AIC	0.649	0.118	0.066	0.058	0.058	0.051	0.713	0.113	0.060	0.040	0.036	0.038
	HQIC	0.807	0.085	0.042	0.029	0.024	0.012	0.874	0.072	0.030	0.010	0.007	0.006
1	*BIC*	0.913	0.057	0.018	0.007	0.004	0.000	0.956	0.035	0.005	0.002	0.001	0.001
	AIC	0.649	0.115	0.069	0.063	0.059	0.046	0.708	0.114	0.063	0.048	0.030	0.037
	HQIC	0.803	0.094	0.039	0.029	0.024	0.011	0.871	0.075	0.026	0.016	0.006	0.006

		$N=200$						$N=400$					
ρ		0	1	2	3	4	5	0	1	2	3	4	5
0.1	*BIC*	0.956	0.034	0.009	0.000	0.000	0.000	0.976	0.020	0.004	0.000	0.000	0.000
	AIC	0.630	0.140	0.094	0.060	0.043	0.032	0.637	0.142	0.096	0.056	0.040	0.029
	HQIC	0.855	0.090	0.036	0.011	0.006	0.002	0.890	0.071	0.028	0.007	0.003	0.001
0.2	*BIC*	0.963	0.030	0.006	0.001	0.000	0.000	0.980	0.017	0.003	0.000	0.000	0.000
	AIC	0.655	0.128	0.087	0.054	0.039	0.037	0.679	0.108	0.084	0.054	0.040	0.034
	HQIC	0.880	0.073	0.028	0.013	0.004	0.002	0.906	0.056	0.025	0.009	0.003	0.001
0.3	*BIC*	0.974	0.019	0.005	0.002	0.000	0.000	0.980	0.016	0.004	0.000	0.000	0.000
	AIC	0.677	0.110	0.073	0.059	0.044	0.037	0.696	0.109	0.070	0.055	0.037	0.033
	HQIC	0.890	0.062	0.029	0.012	0.004	0.002	0.912	0.056	0.021	0.008	0.002	0.001
0.4	*BIC*	0.973	0.019	0.005	0.002	0.000	0.000	0.981	0.017	0.002	0.000	0.000	0.000
	AIC	0.688	0.106	0.070	0.056	0.044	0.036	0.699	0.106	0.068	0.054	0.037	0.035
	HQIC	0.892	0.056	0.028	0.015	0.005	0.003	0.912	0.055	0.018	0.010	0.004	0.001
0.5	*BIC*	0.971	0.023	0.006	0.001	0.000	0.000	0.982	0.017	0.001	0.000	0.000	0.000
	AIC	0.690	0.117	0.062	0.057	0.038	0.035	0.695	0.118	0.062	0.047	0.041	0.036
	HQIC	0.894	0.065	0.021	0.012	0.005	0.003	0.912	0.058	0.020	0.005	0.002	0.003
0.6	*BIC*	0.980	0.017	0.003	0.000	0.000	0.000	0.984	0.013	0.002	0.000	0.000	0.000
	AIC	0.714	0.107	0.061	0.045	0.037	0.035	0.713	0.123	0.063	0.041	0.030	0.030
	HQIC	0.902	0.061	0.019	0.011	0.004	0.003	0.919	0.056	0.016	0.007	0.001	0.002
0.7	*BIC*	0.973	0.024	0.002	0.001	0.000	0.000	0.985	0.015	0.001	0.000	0.000	0.000
	AIC	0.715	0.118	0.063	0.041	0.033	0.030	0.735	0.109	0.061	0.039	0.033	0.023
	HQIC	0.901	0.063	0.019	0.009	0.005	0.003	0.925	0.052	0.013	0.006	0.002	0.001
0.8	*BIC*	0.972	0.023	0.004	0.001	0.000	0.000	0.979	0.020	0.002	0.000	0.000	0.000
	AIC	0.721	0.108	0.064	0.043	0.036	0.028	0.719	0.122	0.060	0.040	0.036	0.023
	HQIC	0.905	0.059	0.019	0.010	0.005	0.002	0.911	0.063	0.017	0.005	0.003	0.001
0.9	*BIC*	0.978	0.020	0.002	0.000	0.000	0.000	0.985	0.013	0.002	0.000	0.000	0.000
	AIC	0.722	0.118	0.061	0.043	0.029	0.026	0.731	0.115	0.061	0.041	0.031	0.021
	HQIC	0.909	0.063	0.018	0.006	0.003	0.001	0.927	0.050	0.014	0.007	0.002	0.000
1	*BIC*	0.972	0.025	0.002	0.001	0.000	0.000	0.984	0.015	0.001	0.001	0.000	0.000
	AIC	0.713	0.126	0.057	0.044	0.032	0.028	0.732	0.112	0.060	0.042	0.028	0.026
	HQIC	0.898	0.069	0.018	0.008	0.005	0.003	0.926	0.051	0.015	0.005	0.002	0.001

Table 11. Relative frequencies of information criteria when an intercept and trend are included in the model

		$N = 50$						$N = 100$					
ρ		0	1	2	3	4	5	0	1	2	3	4	5
0.1	BIC	0.870	0.077	0.038	0.011	0.003	0.001	0.920	0.055	0.020	0.004	0.002	0.000
	AIC	0.515	0.149	0.130	0.082	0.061	0.064	0.555	0.151	0.125	0.072	0.050	0.048
	$HQIC$	0.702	0.126	0.087	0.045	0.022	0.018	0.775	0.108	0.070	0.025	0.012	0.010
0.2	BIC	0.877	0.060	0.039	0.018	0.004	0.001	0.925	0.044	0.024	0.005	0.001	0.001
	AIC	0.539	0.113	0.132	0.090	0.067	0.059	0.586	0.113	0.123	0.078	0.048	0.052
	$HQIC$	0.719	0.097	0.092	0.050	0.026	0.016	0.798	0.086	0.069	0.026	0.012	0.009
0.3	BIC	0.894	0.052	0.034	0.015	0.005	0.001	0.941	0.035	0.018	0.004	0.001	0.001
	AIC	0.566	0.097	0.121	0.094	0.065	0.056	0.611	0.098	0.098	0.085	0.057	0.052
	$HQIC$	0.740	0.087	0.081	0.050	0.023	0.018	0.827	0.070	0.052	0.029	0.012	0.010
0.4	BIC	0.897	0.053	0.031	0.013	0.004	0.002	0.947	0.035	0.012	0.005	0.001	0.000
	AIC	0.588	0.097	0.106	0.084	0.066	0.059	0.633	0.102	0.075	0.079	0.061	0.050
	$HQIC$	0.756	0.082	0.074	0.044	0.027	0.018	0.835	0.072	0.042	0.029	0.014	0.008
0.5	BIC	0.908	0.052	0.025	0.010	0.003	0.001	0.948	0.036	0.011	0.003	0.002	0.000
	AIC	0.600	0.100	0.093	0.079	0.071	0.056	0.640	0.106	0.070	0.067	0.065	0.052
	$HQIC$	0.774	0.083	0.067	0.036	0.025	0.015	0.844	0.072	0.035	0.024	0.017	0.009
0.6	BIC	0.905	0.057	0.025	0.008	0.004	0.001	0.958	0.031	0.007	0.002	0.002	0.000
	AIC	0.613	0.109	0.085	0.073	0.063	0.058	0.668	0.105	0.058	0.059	0.057	0.054
	$HQIC$	0.780	0.094	0.056	0.034	0.025	0.012	0.861	0.070	0.028	0.019	0.015	0.008
0.7	BIC	0.906	0.054	0.027	0.008	0.004	0.001	0.953	0.036	0.008	0.002	0.002	0.000
	AIC	0.611	0.115	0.079	0.070	0.074	0.051	0.674	0.114	0.063	0.050	0.048	0.050
	$HQIC$	0.782	0.092	0.054	0.031	0.027	0.014	0.852	0.080	0.033	0.016	0.012	0.007
0.8	BIC	0.907	0.057	0.021	0.012	0.003	0.001	0.952	0.035	0.008	0.003	0.001	0.000
	AIC	0.627	0.110	0.069	0.068	0.076	0.050	0.671	0.119	0.066	0.049	0.046	0.049
	$HQIC$	0.786	0.089	0.049	0.037	0.028	0.011	0.854	0.077	0.028	0.019	0.013	0.009
0.9	BIC	0.896	0.063	0.026	0.009	0.004	0.001	0.948	0.040	0.009	0.002	0.001	0.001
	AIC	0.610	0.116	0.081	0.070	0.075	0.049	0.679	0.120	0.073	0.047	0.038	0.043
	$HQIC$	0.775	0.098	0.054	0.036	0.028	0.008	0.858	0.082	0.031	0.014	0.010	0.006
1	BIC	0.899	0.059	0.026	0.012	0.003	0.001	0.948	0.040	0.009	0.001	0.001	0.001
	AIC	0.597	0.111	0.084	0.080	0.073	0.055	0.678	0.124	0.069	0.045	0.041	0.044
	$HQIC$	0.763	0.096	0.056	0.045	0.029	0.011	0.853	0.082	0.032	0.015	0.010	0.008

		$N = 200$						$N = 400$					
ρ		0	1	2	3	4	5	0	1	2	3	4	5
0.1	BIC	0.952	0.036	0.011	0.002	0.001	0.000	0.971	0.023	0.006	0.000	0.000	0.000
	AIC	0.579	0.140	0.124	0.070	0.047	0.041	0.613	0.130	0.114	0.063	0.043	0.038
	$HQIC$	0.832	0.086	0.053	0.018	0.007	0.004	0.871	0.066	0.044	0.012	0.004	0.002
0.2	BIC	0.958	0.030	0.010	0.002	0.000	0.000	0.978	0.016	0.005	0.001	0.000	0.000
	AIC	0.618	0.108	0.113	0.072	0.050	0.041	0.643	0.103	0.098	0.071	0.050	0.035
	$HQIC$	0.856	0.067	0.047	0.019	0.007	0.004	0.888	0.056	0.035	0.014	0.005	0.002
0.3	BIC	0.963	0.027	0.008	0.002	0.000	0.000	0.982	0.014	0.004	0.001	0.000	0.000
	AIC	0.636	0.098	0.089	0.080	0.053	0.044	0.657	0.102	0.070	0.077	0.052	0.042
	$HQIC$	0.868	0.061	0.038	0.023	0.007	0.004	0.900	0.051	0.024	0.018	0.005	0.001
0.4	BIC	0.968	0.024	0.006	0.002	0.001	0.000	0.982	0.016	0.002	0.001	0.000	0.000
	AIC	0.659	0.101	0.066	0.073	0.058	0.044	0.674	0.109	0.060	0.062	0.052	0.044
	$HQIC$	0.880	0.060	0.029	0.019	0.009	0.004	0.906	0.052	0.018	0.015	0.007	0.003
0.5	BIC	0.972	0.022	0.004	0.002	0.001	0.000	0.982	0.016	0.002	0.000	0.000	0.000
	AIC	0.679	0.101	0.058	0.055	0.058	0.048	0.698	0.113	0.058	0.040	0.049	0.043
	$HQIC$	0.887	0.062	0.022	0.015	0.010	0.004	0.916	0.052	0.014	0.009	0.006	0.003
0.6	BIC	0.973	0.023	0.004	0.001	0.000	0.000	0.983	0.015	0.002	0.000	0.000	0.000
	AIC	0.696	0.108	0.059	0.043	0.048	0.046	0.710	0.115	0.062	0.036	0.033	0.044
	$HQIC$	0.898	0.064	0.020	0.009	0.006	0.004	0.918	0.055	0.016	0.006	0.004	0.002
0.7	BIC	0.973	0.022	0.004	0.000	0.000	0.000	0.982	0.016	0.002	0.000	0.000	0.000
	AIC	0.706	0.112	0.064	0.046	0.035	0.037	0.728	0.114	0.059	0.039	0.033	0.028
	$HQIC$	0.898	0.063	0.022	0.009	0.006	0.002	0.919	0.056	0.016	0.005	0.002	0.002
0.8	BIC	0.974	0.021	0.004	0.001	0.000	0.000	0.986	0.013	0.001	0.000	0.000	0.000
	AIC	0.712	0.118	0.063	0.045	0.032	0.030	0.725	0.118	0.063	0.039	0.031	0.023
	$HQIC$	0.899	0.065	0.021	0.010	0.004	0.002	0.923	0.053	0.016	0.006	0.003	0.001
0.9	BIC	0.973	0.023	0.004	0.001	0.000	0.000	0.982	0.016	0.002	0.000	0.000	0.000
	AIC	0.724	0.112	0.064	0.041	0.032	0.027	0.728	0.119	0.062	0.040	0.028	0.024
	$HQIC$	0.902	0.064	0.020	0.009	0.003	0.002	0.918	0.057	0.016	0.006	0.002	0.001
1	BIC	0.970	0.026	0.003	0.001	0.000	0.000	0.982	0.016	0.002	0.001	0.000	0.000
	AIC	0.702	0.117	0.070	0.044	0.035	0.032	0.722	0.120	0.062	0.042	0.031	0.025
	$HQIC$	0.889	0.071	0.024	0.010	0.004	0.002	0.918	0.058	0.016	0.007	0.001	0.001

Figure 1: Relative frequencies of information criteria for $\rho = 1$, $N = 100$, model with an intercept and a trend

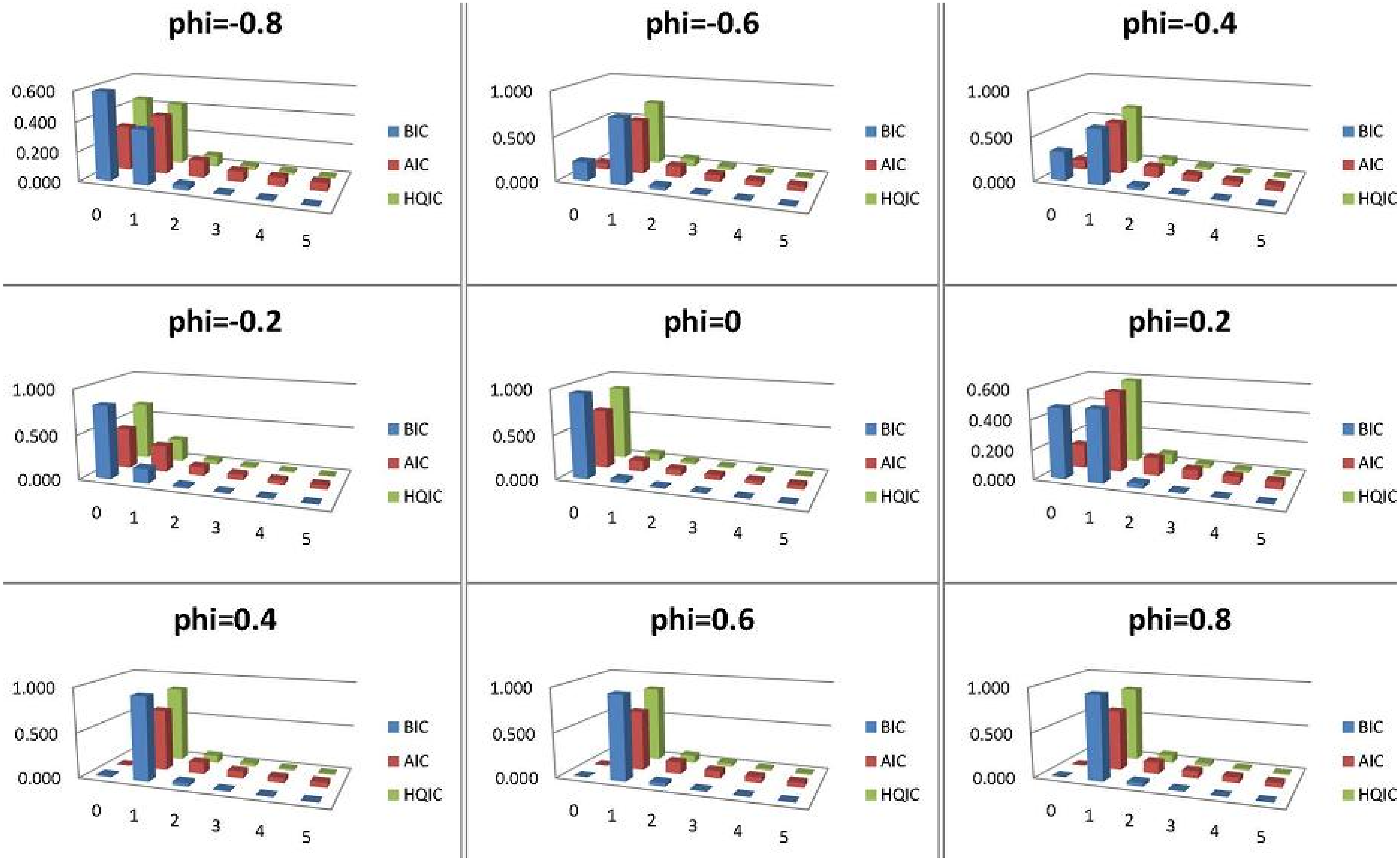


Figure 2: Empirical size of tests for a model with an intercept only

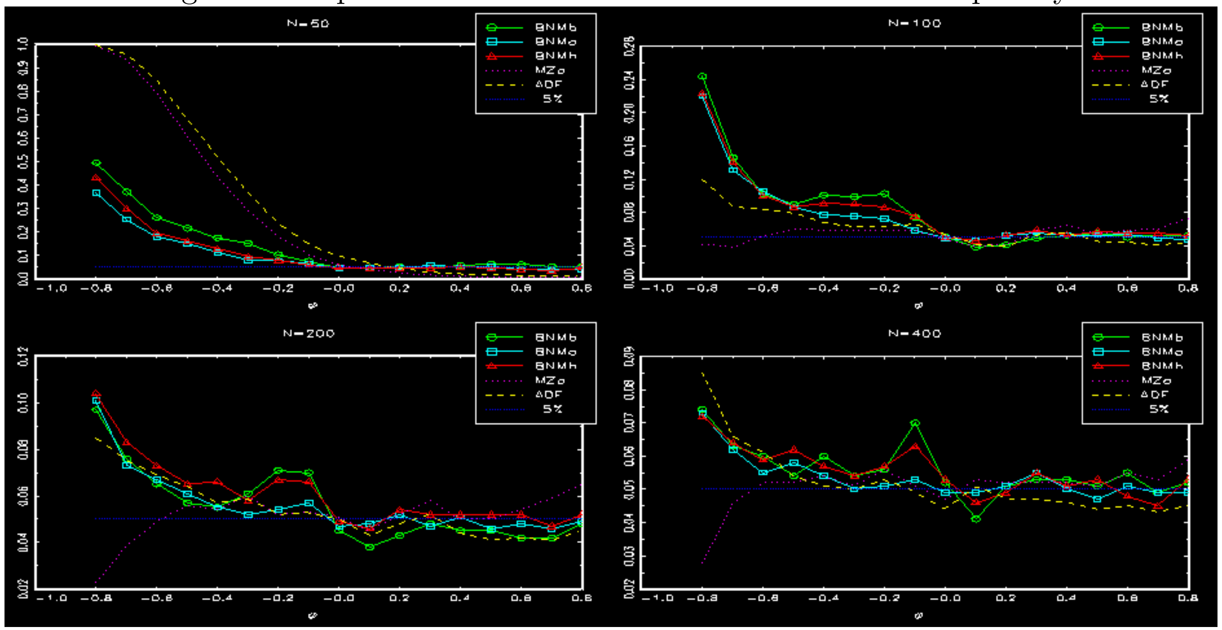


Figure 3: Empirical size for a model with an intercept and a trend

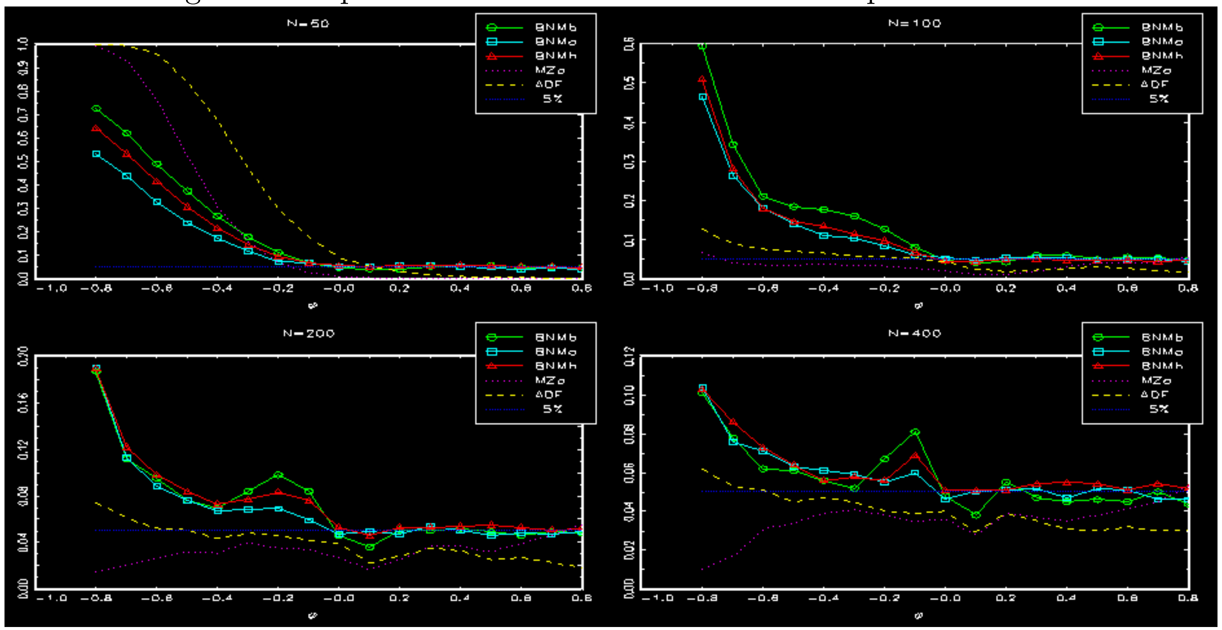


Figure 4: Power for model with an intercept only, $\phi = 0$

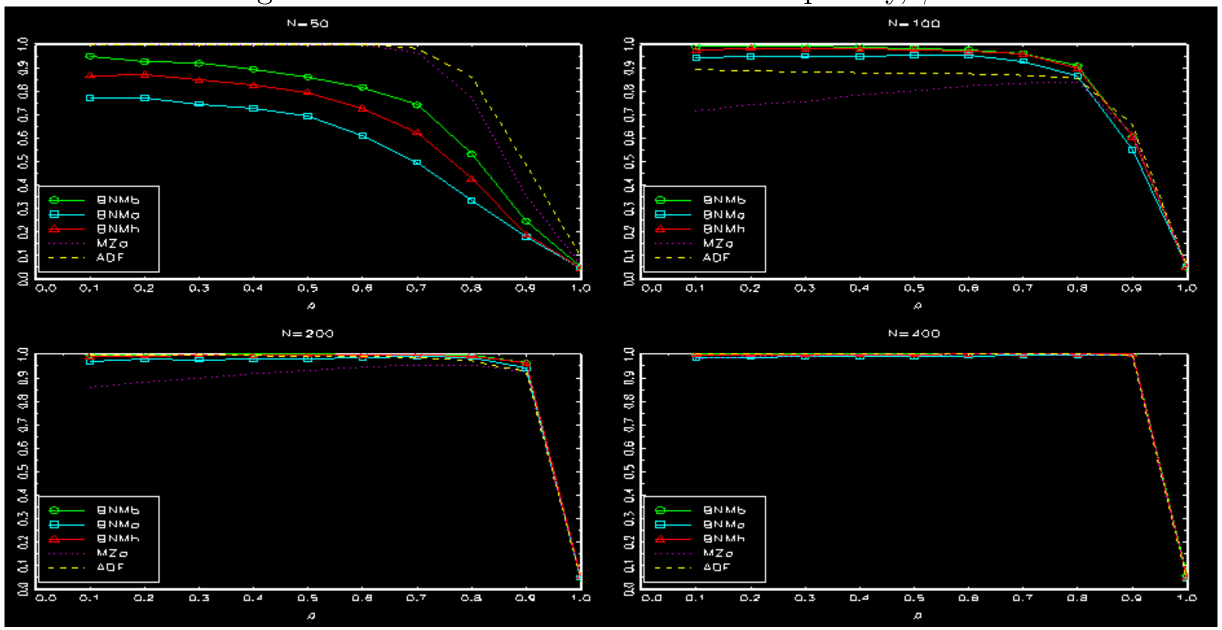
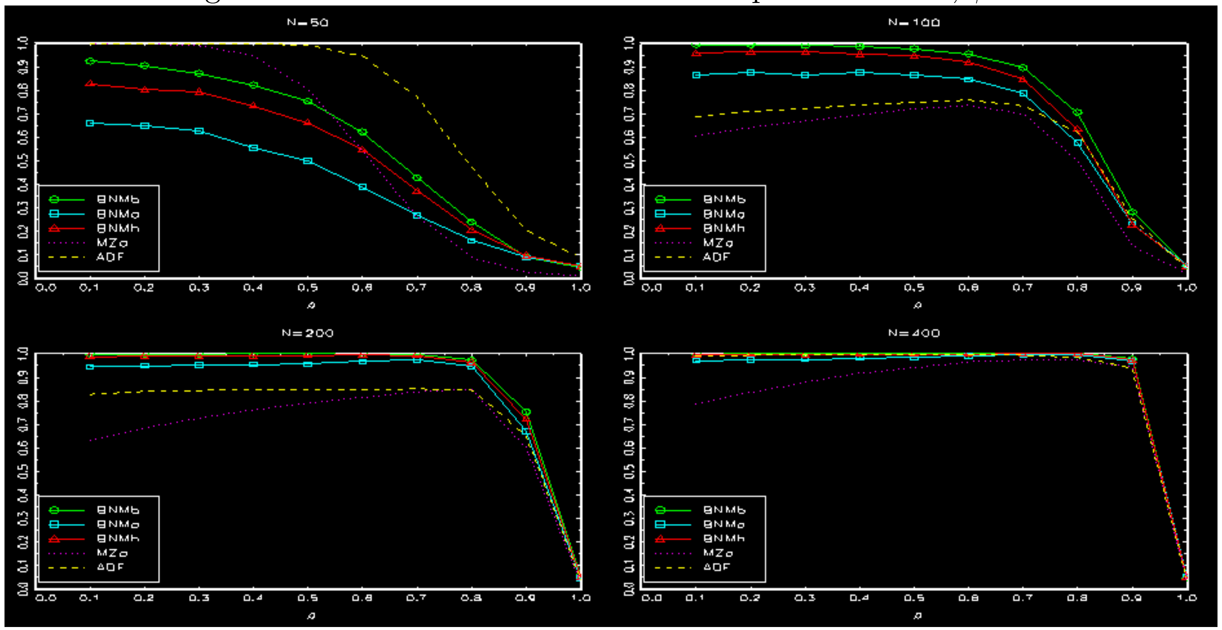


Figure 5: Power for model with an intercept and a trend, $\phi = 0$



10 References

Akaike, H. (1974), A new look at the statistical model identification, *I.E.E.E. Transactions on Automatic Control*, AC 19, 716-723.

Banerjee, A., Dolado, J. J., Galbraith, J. W. and D. F. Hendry (1993), *Co-integration, Error-Correction, and the Econometric Analysis of Non-Stationary Data,* Oxford University Press.

Cox, D. R. and D. V. Hinkley (1974), *Theoretical Statistics,* Chapman and Hall, London.

DeJong, D. N., Nankervis, J. C., Savin, N.E. and C. H. Whiteman (1992), The power problems of unit root tests in time series with autocorrelated errors, *Journal of Econometrics*, 53, pp. 323-343.

Dickey, D. and W. Fuller (1979), Distribution of the estimators for autoregressive time series with a unit root, *Journal of the American Statistical Association*, 74, pp. 427-431.

Dickey, D. and W. Fuller (1981) Likelihood ratio statistics for autoregressive time series with a unit root, *Econometrica,* 49, pp. 1057-1072.

Elliot, G, Rothenberg, T.J. and J.H. Stock (1996), Efficient tests for an autoregressive unit root, *Econometrica,* 64, pp. 813-836.

Engle, R. F. and C. W. J. Granger (1987), Cointegration and error correction: Representation, estimation and testing, *Econometrica,* 51, pp. 251-276.

Forchini, G. F. and P. W. Marsh (2000), Exact inference for the unit root hypothesis, Discussion Paper 00/54, University of York .

Fuller, W. A., (1976), *Introduction to Statistical Time Series,* Wiley, New York.

Granger, C. W. J. and P. Newbold (1974), Spurious regressions in econometrics, *Journal of Econometrics*, 2, pp. 111-120.

Hamilton, J. D., (1994), *Time Series Analysis,* Princeton University Press, New Jersey.

Hannan, E. J., and B. G. Quinn (1979), The determination of the order of an autoregression, *Journal of the Royal Statistical Society, B*, 41, 190-195.

Hillier, G. H., (1987), Classes of similar regions and their power properties for some econometric testing problems, *Econometric Theory*, 3, pp. 1-44.

Horn R.A. and C.R. Johnson, (1985), *Matrix Analysis.* Cambridge University Press, Cambridge

Johansen, S. (1988), Statistical analysis of cointegration vectors, *Journal of Economic Dynamics and Control*, 12, pp. 231-254.

Lehmann, E. L., (1986), *Testing Statistical Hypotheses,* Wiley, New York.

Lehmann, E. L. and C. Stein, (1948), Most powerful tests of composite hypotheses, *The Annals of Mathematical Statistics*, 19, pp. 495-516.

Maddala, G. S. and I. M. Kim (1998), *Unit Roots, Cointegration, and Structural Change,* Cambridge University Press, Cambridge.

Marsh, P. (2005), A measure of discrimination for the unit root hypothesis, Discussion Paper 05/02, University of York .

Marsh, P. (2007), Constructing optimal tests on a lagged dependent variable, *Journal of Time Series Analysis*, 28, 5, pp.723-743.

Nelson, C. R. and C. I. Plosser (1982), Trends and random walks in macroeconomic time series: Some evidence and implications, *Journal of Monetary Economics*, 10, pp. 139-162.

Ng, S. and P. Perron, (2001), Lag length selection and the construction of unit root tests with good size and power, *Econometrica,* 69, 1519–1554.

Perron, P. and Z. Qu (2007), A simple modification to improve the finite sample properties of Ng and Perron's unit root tests, *Economics Letters,* 94, 12–19.

Phillips, P. C. B. (1987), Time series regression with a unit root, *Econometrica,* 55, 277–302.

Phillips, P. C. B. and P. Perron (1988), Testing for a unit root in time series regression, *Biometrika*, 75:2, pp. 335-346.

Plosser, C. I. and Schwert, G. W., (1978), Money, income, and sunspots: Measuring economic relationships and the effects of differencing, *Journal of Monetary Economics*, 4, pp. 637-660.

Said, S. E. and D. A. Dickey (1984), Testing for unit roots in autoregressive-moving average models of unknown order, *Biometrica* 71, pp. 599-607.

Schwarz, G. (1978), Estimating the Dimension of a Model, *Annals of Statistics*, 6, 461-464.

Schwert, G. W., (1989), Tests for unit roots: A Monte Carlo investigation, *Journal of Business & Economic Statistics*, 7:2, pp. 147-159.

Seo, M. H., (2006), Improving unit root testing with a new long run variance estimator, working paper, London School of Economics.

Zaman, A., (1996), *Statistical Foundations for Econometric Techniques,* Academic Press, London.