Additional support questions for all new undergraduate students - ANSWERS

School of Mathematical Sciences, University of Nottingham

September 2021

Section A - Algebraic skills

1. (a) $(x-3)(x+3)$
   (b) $(x-5)(x+3)$
   (c) $(x+14)^2$
   (d) $(8x+3)(x+2)$
   (e) $(x-1)(x-3)(x+5)$
   (f) $(2x-1)(x+2)(x^2+16)$
2. $a=-3$.
3. $p=-4$.
4. $x=9, x=4$.
5. (a) $\dfrac{1}{x+1}-\dfrac{3}{2x-1}$
   (b) $\dfrac{1}{x+1}+\dfrac{3}{x-2}+\dfrac{4}{(x-2)^2}$
   (c) $\dfrac{1-x}{x^2+4}+\dfrac{1}{x-1}$
   (d) $\dfrac{4}{3(x-1)}-\dfrac{2}{3(2x+1)}-\dfrac{1}{(2x+1)^2}$
6. $\dfrac{x-4}{x+5}$

7. $\frac{\sqrt{2}}{2} + \frac{3\sqrt{2}x}{8} + \frac{27\sqrt{2}x^2}{64} + \frac{135\sqrt{2}x^3}{256}$.

8. $9 - 27x + 81x^2$ valid for $|x| < \frac{1}{2}$.

9. $k = -5$ and $n = \frac{1}{3}$.

10. (a) $-2 < x < 12$

    (b) $-\frac{5}{3} \leq x \leq 5$

    (c) $1 < x < 7$

11. $x = \frac{39}{11}, y = \frac{7}{11}$

12. $(-2, -2)$ and $\left(\frac{1}{5}, \frac{23}{5}\right)$.

13. $a = 3$ and $a = 13$.

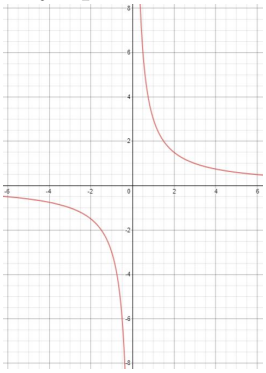
14. $k < 8$.

15. c

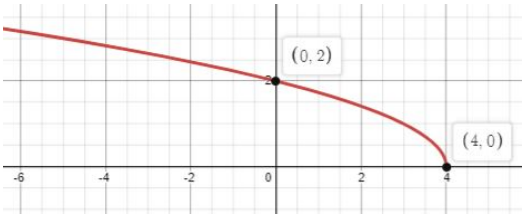
16. xy is not necessarily always positive; y is always negative, but x can be positive, therefore a negative multiplied by a positive gives a negative answer.

Section B - Curve sketching

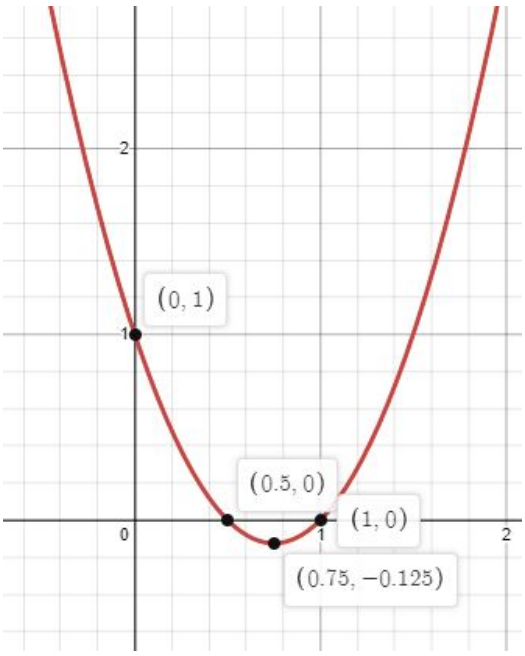
1. (a) Asymptotes at $x = 0$ and $y = 0$



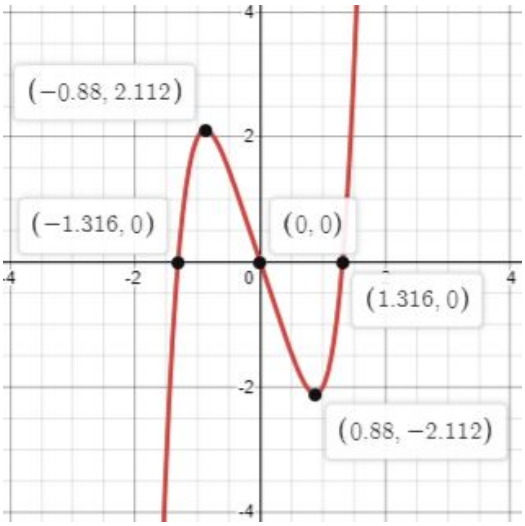
(b)


(0, 2)
(4, 0)
-6
-4
-2
0
2
4

(c)


(0, 1)
(0.5, 0)
(1, 0)
(0.75, −0.125)
2
1
0
1
2

(d)


(−0.88, 2.112)
(−1.316, 0)
(0, 0)
(1.316, 0)
(0.88, −2.112)
4
2
0
-2
-4
-4
-2
4

2. (a) $f(x-3) = \sqrt{x-3}$. The curve is translated by 3 units in the positive x direction.

   (b) $f(x)+2 = \sqrt{x}+2$. The curve is translated by 2 units in the positive y direction.

   (c) $4f(x) = 4\sqrt{x}$. The graph is stretched by a scale factor of 4 parallel to the y axis.

3. (a) $f(x+1) = x^2 - x$. The curve has been translated by 1 unit in the negative x direction.

   (b) $-f(x) = -x^2 + 3x - 2$. The curve has been reflected in the line $y = 0$.

   (c) $f(2x) = 4x^2 - 6x + 2$. The curve has been stretched horizontally by a scale factor of $\frac{1}{2}$.

Section C - Differentiation

1. (a) $\dfrac{1}{\sqrt{2x-5}}$

   (b) $e^{4x}(4\cos 3x - 3\sin 3x)$

   (c) $u'v + uv'$

   (d) $\dfrac{(2x-\cos x)\cos 2x + (x^2 - \sin x)2\sin 2x}{\cos^2 2x}$

   (e) $\dfrac{4}{3}\left(1 - \dfrac{1}{x^2}\right)\left(x + \dfrac{1}{x}\right)^{\frac{1}{3}}$

   (f) $\dfrac{u'v - uv'}{v^2}$

   (g) $\dfrac{e^{\frac{x}{2}}(5-2x)}{2(3-2x)^{\frac{3}{2}}}$

   (h) $f'(x)g(x) + f(x)g'(x)$

   (i) $-3xe^{1-5x}(5x-2)$

   (j) $f'(g(x))g'(x)$

   (k) $\dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

2. $\dfrac{dy}{dx} = \dfrac{15}{(2-5x)^2}$. At the point where $x = 0.2$, $\dfrac{dy}{dx} = 15$

3. $\dfrac{dy}{dx} = \dfrac{4-9x}{2\sqrt{2-3x}}$. At a stationary point $\dfrac{dy}{dx} = 0$, therefore the stationary point is $\left(\dfrac{4}{9}, \dfrac{4\sqrt{6}}{27}\right)$.

4. $y = -x - \frac{1}{2} + \ln 2$.

5. $\dfrac{dy}{dx} = \dfrac{2y}{6y^3 + 5\sqrt{y}}$.

6. $\dfrac{dy}{dx} = 2\cos 2x$, $\dfrac{d^2y}{dx^2} = -4\sin 2x$, therefore $\dfrac{d^2y}{dx^2} = -4y$.

7. (a) $\dfrac{dy}{dx} = \dfrac{-y^4}{3y^3x - 5}$

   (b) $\dfrac{dy}{dx} = \dfrac{2+2y}{3-2x}$

8. $2y = x - 11$.

9. Maximum at $\left(\sqrt{\dfrac{38}{33}}, 2\sqrt{\dfrac{114}{11}}\right)$, minimum at $\left(-\sqrt{\dfrac{38}{33}}, -2\sqrt{\dfrac{114}{11}}\right)$.

Section D - Integration

1. (a) $-\dfrac{2}{9}(4-3x^2)^{\frac{3}{2}} + c$

   (b) $\sqrt{3} - \dfrac{2}{3}$

   (c) $\dfrac{1}{2}\left(x + \dfrac{1}{2}\sin 2x\right) + c$

   (d) $-\dfrac{1}{3}\cos(x^3) + c$

   (e) 2π

   (f) $x\ln x - x + c$

(g) $\frac{1}{8}(\cos 8x - 4\cos 2x) + c$

(h) $\ln 7$

2. $a = \frac{52}{3}$

3. $\frac{20012\pi}{15}$

4. $\frac{27\pi}{80}$

Section E - Trigonometry

1. LHS= $\frac{1}{\sin 2\theta} + \frac{\cos 2\theta}{\sin 2\theta} = \frac{1 + \cos 2\theta}{\sin 2\theta} = \frac{1 + 2\cos^2\theta - 1}{2\sin\theta\cos\theta} = \cot\theta$=RHS

2. $\theta = -\frac{11\pi}{6}, -\frac{3\pi}{2} - \frac{7\pi}{6}, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

3. $y = 6\cos x + 8\sin x = 10\sin(x + 0.644)$.
   Maximum: $\left(\frac{\pi}{2} - 0.644 \pm 2n\pi, 10\right)$
   Minimum: $\left(\frac{3\pi}{2} - 0.644 \pm 2n\pi, -10\right)$

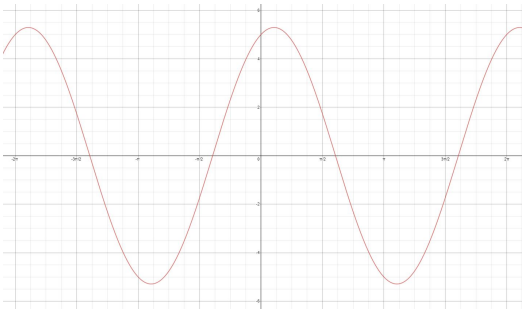
4. LHS= $(\cos^2 x)^2 - (\sin^2 x)^2 = \left(\frac{1}{2}(\cos 2x + 1)\right)^2 - \left(\frac{1}{2}(1 - \cos 2x)\right)^2 =$
   $\left(\frac{1}{4}(\cos^2 2x + 2\cos 2x + 1)\right) - \left(\frac{1}{4}(\cos^2 2x - 2\cos 2x + 1)\right) = \frac{1}{4}\cos^2 2x +$
   $\frac{1}{2}\cos 2x + \frac{1}{4} - \frac{1}{4}\cos^2 2x + \frac{1}{2}\cos 2x - \frac{1}{4} = \cos 2x$ =RHS

5. $\frac{5}{2}\cos 2x - \frac{1}{2}$

6. (a) LHS= $\frac{\sin^2 x + (1 - \cos x)(1 + \cos x)}{(1 + \cos x)\sin x}$
   $= \frac{\sin^2 x + 1 - \cos^2 x}{(1 + \cos x)\sin x} = \frac{2\sin^2 x}{(1 + \cos x)\sin x}$
   $= \frac{2\sin x}{1 + \cos x} = \frac{4\sin\frac{1}{2}x\cos\frac{1}{2}x}{2\cos^2\frac{1}{2}x} = \frac{2\sin\frac{1}{2}x}{\cos\frac{1}{2}x} = 2\tan\frac{1}{2}x$ =RHS

(b) LHS=$\tan(2\theta+\theta) = \dfrac{\tan 2\theta + \tan\theta}{1-\tan 2\theta\tan\theta} = \dfrac{\dfrac{2\tan\theta}{1-\tan^2\theta}+\tan\theta}{1-\tan\theta\times\dfrac{2\tan\theta}{1-\tan^2\theta}} =$

$\dfrac{2\tan\theta+\tan\theta(1-\tan^2\theta)}{(1-\tan^2\theta)-\tan\theta\times 2\tan\theta} = \dfrac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}$ =RHS

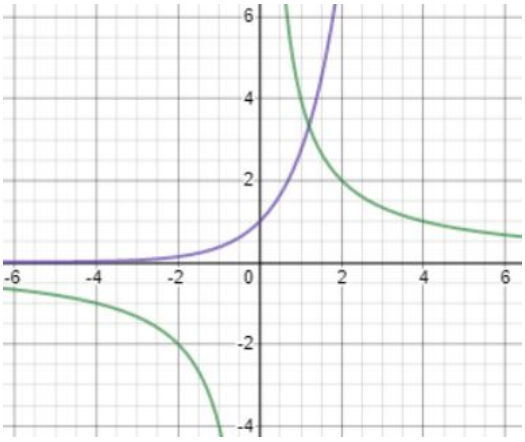
7. 

8. (a) $90°, 210°, 330°$

   (b) $-135°, -104°, 45°, 76°$

Section F - Numerical methods

1. When $x=-1$, $e^x-3x^2<0$, when $x=0$, $e^x-3x^2>0$ therefore a sign change indicates a root.
   When $x=0$, $e^x-3x^2>0$, when $x=1$, $e^x-3x^2<0$ therefore a sign change indicates a root.
   When $x=3$, $e^x-3x^2<0$, when $x=4$, $e^x-3x^2>0$ therefore a sign change indicates a root.
   Roots are: $x=-0.46, x=0.91, x=3.73$

2. 

   $x = 1.202$
3. 3.923736
4. 0.52831

Section G - Parametric equations

1. (a) $y = \dfrac{x^2}{2} - x + \dfrac{1}{2}$

   (b) $\dfrac{x^2}{4} + \dfrac{y^2}{64} = 1$

   (c) $y = \dfrac{x+8}{2x+1}$
2. $8\sqrt{5}$
3. $y = \dfrac{3\sqrt{3}x}{2} - \dfrac{5\sqrt{3}}{4}$
4. Minimum at $(1, -2)$

Section H - Differential equations

1. (a) $y = \dfrac{A}{(1-5x)^{\frac{3}{5}}}$

   (b) $y = Ax - 5$

2. (a) $3y^2 + 18y = 2x^3 + 5$

   (b) $y = -\dfrac{4}{x^2} + 3$

3. 7.3 seconds

4. During the year 2023

5. $y = \dfrac{1}{2} \ln \dfrac{x+1}{x+3} + c$

6. $y = 10\sqrt{x} + 3$

Section I - Probability and Statistics

1. (a) Mean= £91.16, standard deviation= 26.10

   (b) Students should describe the hypotheses that is being tested, e.g. null hypothesis and alternative hypothesis. They should also mention that it is a 1 sided test. Mention how we decide on the rejection zone.

2. 28.75

3. $P(X \geq 3) = 0.01585$

4. $P(A \cup B) = \dfrac{7}{30}$

5. $P(N \geq 2) = \dfrac{9}{13}$

6. $P(W \cap R) = \dfrac{8}{25}$

7. 13 potatoes

Section J - Extension and enrichment

1. $1 < x < 3$ or $-3 < x < -1$

2. $f(x) = -\dfrac{5}{2} - \dfrac{7}{4}x - \dfrac{17}{8}x^2 - \dfrac{31}{16}x^3 + ...$ valid for $|x| < 1$

3. The fishtank is 50cm by 30cm by 30cm
4. One real root
5. $\dfrac{d}{dx}\tan x = 1 + \tan^2 x.$
6. $f'(a) = 0.$
7. $v = \dfrac{\pi r^2 h}{3}$
8. $-\cos x + \dfrac{\cos^3 x}{3} + c$
9. $x = 41.4°, 60°, 75.5°, 284.5°, 300°, 318.6°$
10. $\left(\dfrac{5}{3}\pi, \dfrac{1}{2}\right), \left(\dfrac{11}{3}\pi, \dfrac{1}{2}\right), \left(\dfrac{17}{3}\pi, \dfrac{1}{2}\right), \left(\dfrac{23}{3}\pi, \dfrac{1}{2}\right)$